OTTAWA RIVER NUCLEAR SPILL CONTINGENCY MODEL DEVELOPMENT

TECHNICAL REPORT PREPARED FOR

THE ONTARIO MINISTRY
OF THE ENVIRONMENT

GORE & STORRIE LIMITED
1670 BAYVIEW AVENUE
TORONTO, ONTARIO
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LETTER OF TRANSMITTAL

15 November 1984

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ONTARIO MINISTRY OF THE ENVIRONMENT
1 St. Clair Avenue West
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Toronto, Ontario M4V 1P5

Attention: Mr. Dennis Draper

Gentlemen:

Re: Ottawa River Nuclear Spill Contingency Model Development

We are pleased to submit our final report for the above-noted study. The salient aspects of the study are described in the Executive Summary.

The first chapter presents a background to the study and outlines the objectives. The following chapter deals with theoretical aspects of 2-D finite-time model development including description of the computer program, TWODIFIN, and an illustrative example. Analyses related to convective period and linking of near-field and far-field models are presented in the third chapter. The Appendices A and B include details of some mathematical derivations and documentation for the computer program TWODIFIN.

We wish to thank the Ministry for giving us the opportunity to undertake this research and development project.

All of which is respectfully submitted.

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Technical Report Prepared For
The Ontario Ministry Of The Environment

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EXECUTIVE SUMMARY

As part of the Ottawa River nuclear spill contingency planning studies, a model which will aid in predicting the arrival times, durations of passage, and concentration distributions of the spilled radionuclide at the water supply intakes and other strategic locations has been developed. The model utilizes an analytical solution for the two-dimensional finite time case, the derivation of which is based on the stream tube concept wherein the lateral co-ordinate is represented by the cumulative discharge. An approximate integration procedure, known as the Laplace method, has been utilized in the derivations. The analytical solution applicable to pipe outfalls includes the effects of side-wall reflection of the material from channel banks. Expressions have been presented for the expected arrival times and durations of passage of spilled material cloud at strategic locations.

A model with the acronym, TWODIFIN, has been developed to perform the computations. The model accounts for longitudinal variabilities of channel geometric and hydraulic parameters (viz, width, depth, velocity, longitudinal dispersion coefficient, and lateral diffusion factor) and radionuclide decay rates. This is achieved by dividing the channel study segment into a number of reaches such that the transport process parameters are fairly constant within each reach and by utilizing the moving average values of the parameters in the computations. The concentrations and flow rates of the spill discharge are specified as step inputs occurring over a known number of time intervals. The other inputs to the program include the channel hydraulic parameters, longitudinal dispersion coefficients, lateral diffusion factors, and radionuclide decay rates for successive reaches of river channel below the outfall. The output of the model includes lateral concentration distributions of radionuclide material as a function of time at specified locations below the outfall. The times of prediction are generated from the calculated arrival times and durations of passage.

An example based on the data gathered in the Ottawa River following a spill of tritium from the Rolphton nuclear power demonstration (NDP) facility during 1981 illustrates the input parameters and output summary tables of the TWODIFIN model. It should be noted that this example is an illustration of the computational aspects; it is not intended for a validation of the model.

The importance of the convective period in the Ottawa River below the Rolphton NPD discharge has been assessed with the aid of the tritium concentration-versus-time distributions monitored at Deep River, Petawawa and Pembroke during August - September 1981, following the tritium spill from the Rolphton NPD in August 1981. The convective period is found to dominate for about 16-20 km below Rolphton as indicated by a skewed concentration distribution at Deep River; however, the degree of skewness relative to that for a normal distribution (skewness = 0) is found to be small. Thus, it is concluded that the effect of convective period can be neglected in the modelling studies. It is recommended that this aspect be further examined with the aid of data from the recent field studies (conducted during September 1984) and modelling studies.

The relation between the convective period and lateral mixing zones has been examined. A detailed assessment could not be made because of a lack of relevant information (either theoretical or field study data); it is recommended to further evaluate this aspect with the aid of 2-D model and the field data gathered in the recent field studies.

Methods for linking near-field and far-field models have been presented. A schematic representation shows the various zones in the near-field and far-field regions, and the models applicable to these zones. The methods for linking of models presented herein will permit a continuity in applying the models for the near-field and far-field regions.

SOMMAIRE

L'élaboration du plan d'urgence en cas de déversement nucléaire dans la rivière des Outaouais nous a amenés à mettre au point un modèle qui aidera à prédire l'arrivée des radionucléides aux prises d'eau et à d'autres points stratégiques, leur durée de transit et la distribution de leurs concentrations. Le modèle utilise une solution analytique pour décrire l'incident bidimensionnel à temps fini, solution reposant sur le concept des tubes d'écoulement dans lequel la coordonnée latérale est représentée par le déversement cumulatif. Pour parvenir aux dérivées, nous nous sommes servis d'une technique d'intégration approximative connue sous le nom de méthode de Laplace. La solution analytique qui s'applique aux points de déversement tient compte des nucléides qui sont reflétés des berges du chenal. Des expressions ont été formulées pour l'arrivée estimative du nuage en des endroits clés ainsi que sa durée probable.

Un modèle portant l'acronyme TWODIFIN a été élaboré pour effectuer les calculs. Ce modèle rend compte des variabilités longitudinales des paramètres géométriques et hydrauliques du chenal (c'est-à-dire la largeur, la profondeur, la vitesse, le coefficient de dispersion longitudinale et le facteur de diffusion latérale) ainsi que des taux de désintégration des radionucléides. Il y arrive en divisant la partie du chenal à l'étude en un certain nombre de tronçons dans lesquels les paramètres du déplacement sont assez constants, puis en utilisant les valeurs moyennes du déplacement dans les calculs. Les concentrations et les taux d'écoulement des substances déversées sont considérés comme des entrées qui se produisent à un nombre connu d'intervalles. Les autres entrées sont les paramètres hydrauliques du chenal, les coefficients de dispersion longitudinale, les facteurs de diffusion latérale et les taux de désintégration des radionucléides pour les tronçons successifs du chenal de la rivière en aval du point de déversement. Quant aux sorties, elles comprennent la distribution des concentrations de radionucléides, en fonction du temps, à des endroits précis en aval du point de déversement. Les prédictions temporelles proviennent du moment d'arrivée et de la durée de transit, déjà calculés.

Un exemple qui s'inspire des données recueillies dans la rivière des Outaouais par suite du déversement de tritium survenu en 1981 à la centrale nucléaire pilote de Rolphton illustre bien les paramètres des entrées et les tableaux récapitulatifs des sorties du modèle TWODIFIN. Il faut faire remarquer que cet exemple a pour seul but d'illustrer les calculs, et non de vérifier le modèle.

L'importance de la période de convection dans la rivière des Outaouais en aval du déversement survenu à la centrale de Rolphton a été évaluée au moyen des concentrations de tritium, en fonction du temps, qui ont été mesurées à Deep River, à Petawawa et à Pembroke en août et en septembre 1981, après l'incident, qui s'est produit en août de la même année. La période de convection, avons-nous constaté, domine sur une distance approximative de 16 à 20 kilomètres en aval de Rolphton, comme le montre la distribution asymétrique des concentrations à Deep River; toutefois, le degré d'asymétrie est minime si on le compare à celui d'une distribution normale (valeur 0). Ainsi, nous concluons qu'il n'est pas nécessaire de tenir compte des effets de la période de convection dans la réalisation des modèles. À cet égard, nous recommandons que cet aspect soit examiné plus avant en se servant des données provenant des études réalisées sur le terrain en septembre 1984 et des études de modélisation.

La relation qui existe entre la période de convection et les zones de mélange latérales a été étudiée. Une évaluation détaillée s'est avérée impossible à cause du manque de renseignements, soit théoriques, soit pratiques; nous proposons donc que cet élément soit approfondi au moyen du modèle bidimensionnel et des données recueillies au cours des études sur le terrain.

Les moyens de lier les modèles du champ proche et les modèles du champ éloigné ont été présentés. La représentation schématique indique les diverses zones qui se trouvent dans les deux champs, ainsi que les modèles qui s'y appliquent. Ces méthodes de liaison des modèles permettront d'assurer la continuité d'application de ceux-ci dans le cas et des champs proches et des champs éloignés.

ACKNOWLEDGEMENTS

Thanks are due to Mr. Dennis Draper, MOE Project Liaison Officer, for providing the necessary data, and for constructive discussions during the course of this study.

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I INTRODUCTION

1.1 Background

Contingency planning studies undertaken by the Ontario Ministry of the Environment (MOE) for the management of accidental spills or controlled release of radionuclides into the Ottawa River from nuclear power plants include development of mathematical models having the ability to predict the concentrations and durations of passage of such radionuclide releases at municipal water intakes and other strategic locations. As a part of this study, methods for estimating the longitudinal dispersion coefficients as well as an analytical equation for the one-dimensional finite-time time release case, i.e. transient 1-D model, have been presented in a previous report (Gore & Storrie, Ltd., 1983). Some of the aspects requiring further studies which were identified in that report include development of a far-field finite-time 2-D model, methods for linking near and far-field models, assessing the importance of the convective period and evaluation of the relation between the convective period zone and lateral mixing zone.

A finite-time 2-D model is required because of the fact that the discharge at the source is either from a pipe or a diffuser outfall which usually results in lateral concentration gradients being set up for a considerable downstream distance. For example, the distance required for complete lateral mixing downstream from the Rolphton NPD is estimated to be nearly 43 km; on the other hand, vertical mixing is estimated to take place in less than 1 km (Gore & Storrie, Ltd., 1983). It was also estimated that the 1-D bulk diffusion equations, valid in the diffusive period, are applicable after a convective period which dominates for about 50 km below the Rolphton NPD. The length of the convective period zone was estimated from an equation developed by Fischer (1967); however, this was not assessed with the aid of concentration distributions. These estimates also indicate that the zones of convective period and lateral mixing extend for almost the same length of the river, which points out a need to assess their relative significance in the context of modelling studies. Methods for linking near-field and far-field models are also needed to establish continuity among near-field, far-field 2-D and far-field 1-D zones in modelling studies.

1.2 Objectives

The objectives of this investigation are as follows:

1. To develop a far-field 2-D finite-time model, which includes longitudinal advection, dispersion and decay, and lateral dispersion.

2. To present methods for linking near-field and far-field models.

3. To assess the importance of the "convective period" in the Ottawa River below the Rolphton NPD.

4. To evaluate the relationship between the convective period zone and the lateral mixing zone.

II FAR-FIELD 2-D TRANSIENT MODEL

2.1 Theoretical Aspects

2.1.1 Differential Equation

The basic differential equation describing the conservation of mass for the 2-D transient case can be obtained by following the procedures presented by Yotsukura and Sayre (1976). The differential equation, based on the "stream tube" concept wherein the transverse co-ordinate is represented by the cumulative discharge (q) can be written in the following form:

$$\frac{\partial c}{\partial t} + \frac{u_x}{m_x}\frac{\partial c}{\partial x} = \frac{e_x}{m_x}\frac{\partial^2 c}{\partial x^2} + \frac{uD_y}{m_x}\frac{\partial^2 c}{\partial q^2} - K_d c \qquad (1)$$

where x = longitudinal co-ordinate (m)

q = transverse co-ordinate (m^3/s)

t = time (s)

u_x = velocity in the x-direction (m/s)

m_x = metric coefficient (or scaling factor) associated with x-axis

e_x = longitudinal dispersion coefficient (m^2/s)

D_y = transverse diffusion factor (m^5/s^2)

K_d = first-order decay rate coefficient (1/s)

c = concentration at a point (x, q) at time (t)

Equation 1 is subject to the following assumptions:

1. The density of effluent and receiving water are the same.

2. The concentration distributions in the far-field region are not affected by the near-field processes such as the jet-effect.

3. The vertical concentration distributions have attained uniformity just below the outfall.

4. The decay of pollutant follows the first-order kinetic law.

The independent variable (q) and the diffusion factor (D_y) appearing in Equation 1 are defined by:

$$q = \int_0^y m_y\, u_y\, h_y \cdot dy \qquad (2)$$

$$D_y = \overline{m_x\, e_y\, u_y\, h_y^2} = \frac{1}{Q}\int_0^Q m_x\, e_y\, u_y\, h_y^2 \cdot dq \qquad (3)$$

wherein the overbar indicates the average of the product term ($m_x\, e_y\, u_y\, h_y^2$); m_y is a metric coefficient associated with the y-axis; h_y and u_y denote, in order, the local depth and local depth-averaged velocity in the x-direction at a lateral distance (y) w.r.t a reference bank of channel; e_y = transverse dispersion coefficient; and Q = discharge in the river channel just below the source. The metric coefficients, m_x and m_y, account for the convergence or divergence of stream surfaces along the respective axes, x and y. The diffusion factor (D_y) can also be expressed by (Gowda, 1980; 1984a; 1984b):

$$D_y = \overline{m}_x\, \Psi\, e_y\, \overline{u}\, \overline{h}^2 = Q^2 \beta / b \qquad (4)$$

where $\overline{h}$ and $\overline{u}$ denote, in order, the cross-sectional mean values of depth and velocity; b = channel width; Ψ = shape-velocity factor; β = nondimensional diffusion factor; and $\overline{m}_x$ = average value of m_x.

Methods for estimating e_x, D_y, β, Ψ, m_x and m_y have been described in various publications (Yotsukura and Cobb, 1972; Beltaos, 1979; Gowda, 1980; Gore & Storrie Ltd., 1983). It should be noted that the expressions for β and D_y incorporate Ψ, m_x and m_y. A summary of the values of β for natural streams can be found in other publications (Gowda, 1981; 1984a). The decay rate coefficient (K_d) at a known temperature (T_1°C) for a given pollutant may be obtained from field study data or a literature review. The rate coefficient at a design temperature (T_2°C) is calculated from the modified van't Hoff-Arrhenius relationship:

$$K_{T2} = K_{T1}\,\theta^{T_2 - T_1} \qquad (5)$$

in which K_{T1} and K_{T2} denote the rate coefficients at T_1° and T_2° C, respectively; and θ is the temperature correction factor.

2.1.2 Analytical Solution:

For the sake of convenience in obtaining analytical solutions, Equation 1 is rewritten as follows:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = E_x \frac{\partial^2 c}{\partial x^2} + E_q \frac{\partial^2 c}{\partial q^2} - K_d c \qquad (6)$$

where

$$u = \frac{u_x}{m_x}\ ; \qquad E_x = \frac{e_x}{m_x^2}\ ; \qquad E_q = \frac{uD_y}{m_x} \qquad (7)$$

Although m_x occurs in these equations, it can be assumed to have a unit value. This approximation is not likely to have a major effect on the computations.

An analytical solution of Equation 6 for the case of an instantaneous pollutant source located at $x = 0$ and $q = q_s$ (with respect to origin at a reference bank of channel) is given by:

$$c_I = \frac{W_I u}{2\pi t\sqrt{E_x E_q}} \exp\left\{ - \frac{(x - ut)^2}{4E_x t} - \frac{(q_s - q)^2}{4E_q t} - K_d t \right\} \qquad (8)$$

in which q_s = cumulative discharge between the reference bank and the source; W_I = total mass of material discharged instantaneously; and C_I = concentration at a point (x, q) at time (t) due to W_I.

The terms occurring inside the square brackets in Equation 8 can be simplified as outlined by Glover (1964) and Wnek and Fochtman (1972) to obtain a relationship:

$$c_I = \frac{W_I u.\exp\,(ux/2e_x)}{2\pi\sqrt{E_x E_q}} \quad \frac{\exp\left(-\frac{\zeta}{t} - \eta t\right)}{t} \tag{9}$$

$$\text{where} \quad \zeta = \left\{\frac{x^2}{4E_x} + \frac{(q_s - q)^2}{4E_q}\right\} \quad ; \qquad \text{and} \quad \eta = \left\{\frac{u^2}{4E_x} + K_d\right\} \tag{10}$$

The concentration distribution due to the release of a neutral density material at a rate (w) at the point x = 0 and q = 0 during a finite time (dt') extending from $t' = t_o$ to $t' = t_n$ can be obtained by integrating Equation 9 with respect to time. Thus, for the finite-time case, we have:

$$c = \frac{u.\exp\,(ux/2e_x)}{2\pi\sqrt{E_x E_q}} \int_{t_o}^{t_n} \frac{w.dt'}{(t - t')} \cdot \exp\left\{-\left(\frac{\zeta}{t - t'}\right) - \eta\,(t - t')\right\} \tag{11}$$

The rate at which the material is released (w) may vary over the duration of injection ($t_n - t_o$). In such cases, the duration ($t_n - t_o$) can be divided into a number (N) of infinitesimally small time intervals such that within each infinitesimal interval ($t_j - t_i$), the injection rate (Δw) is fairly constant. Then, the concentration (Δc) at a point (x, q) at a given time (t) due to the constant rate injection during ($t_j - t_i$) is given by:

$$\Delta c = \frac{u.\exp\,(ux/2e_x)}{2\pi\sqrt{E_x E_q}} (\Delta w) \int_{t_i}^{t_j} \frac{dt'}{(t - t')} \cdot \exp\left[-\left(\frac{\zeta}{t - t'}\right) - \eta\,(t - t')\right] \tag{12}$$

The total concentration (c) at the point (x, q) at the time (t) due to the material released over the entire duration $(t_n - t_o) = N\,(t_j - t_i)$ can be obained by summing the individual Δc values. Thus:

$$c = \sum_{1}^{N} \Delta c = \frac{u.\exp\left(\frac{ux}{2e_x}\right)}{2\pi\sqrt{E_x E_q}} \sum_{k=1}^{N} \left[(\Delta W_k) \int_{t_i}^{t_j} \frac{dt'}{(t - t')} \cdot \exp\left\{ - \left(\frac{\zeta}{t - t'}\right) - \eta\,(t - t')\right\} \right] \quad (13)$$

The time integral appearing in Equation 13 can be evaluated approximately by a procedure known as the Laplace method (Erdelyi, 1956), the details of which can be found in Appendix A. This method has been utilized to solve 2-D and 3-D equations by Henry and Foree (1979) and Sagar (1982). However, the limits of integration in these two studies have been taken to be $t_i = 0$ and $t_j = t$ (i.e., from 0 to t), the upper limit of which coincides with the time (t) at which the concentration at a point (x, q) is calculated. Wnek and Fochtman (1972) have presented analytical solutions for the 3-D case without resorting to the approximate Laplace method, but by a direct integration procedure using the integration limits, 0 to t. Obviously, the use of integration limits 0 to t is inconsistent with the solution being sought. Concentrations calculated from those analytical solutions are either -ve or unrealistically large. Medina and Buzun (1981) obtained analytical solutions for the 1-D case by direct integration using the limits 0 to t_1. Such a procedure was also the basis of development of analytical solution for the finite-time 1-D case in the Ottawa River Study (Gore & Storrie Ltd., 1983); however, it was pointed out to evaluate the need for changing the integration limits to t_1 to t_2. These solutions (i.e., with integration limits 0 to t_1) are also found to predict unreasonable concentration values. Based on these analyses, the integration limits t_i to t_j are utilized in the formulations presented in this report.

Evaluation of the integral in Equation 13 by the Laplace method (Section A.1, Appendix A) will result in the following expression for the concentration distribution:

$$c = \frac{u.\exp\left(\frac{ux}{2e_x}\right)}{4\sqrt{\pi E_x E_q}} \sum_{1}^{N} \left\{ (\Delta W) \left[\frac{\exp\left(-2\sqrt{\zeta\eta}\right)}{(\zeta\eta)^{1/4}} \left\{ \operatorname{erf}\left(\sqrt{\frac{\zeta}{T_j}} - \sqrt{\eta T_j}\right) - \operatorname{erf}\left(\sqrt{\frac{\zeta}{T_i}} - \sqrt{\eta T_i}\right)\right\}\right]\right\} \quad (14a)$$

$$\text{where} \quad T_i = (t - t_i) \quad \text{and} \quad T_j = (t - t_j). \quad (14b)$$

2.1.3 Analytical Solution with Side-Wall Reflection Effects:

Equation 14 does not include the effects of side-wall reflection of the material from the channel banks. Following Yotsukura and Cobb (1972), let us consider that the material is released as a point (or vertical line) source at a location $x = 0$ and $q = q_s$. The solution which accounts for the side-wall reflection of the material is then obtained from the method of images. For the sake of convenience in writing the resulting solution in a compact form, the following terms are defined (assuming $m_x = 1$):

$$\eta = \left(\frac{u^2}{4e_x} + K_d\right) ; \qquad \alpha = \frac{x^2}{4e_x} ; \qquad E_q = \frac{\beta Q^2 u}{b} \tag{15a}$$

$$p = q/Q ; \qquad p_s = q_s/Q \tag{15b}$$

$$\Psi_1 = b(2n + p_s - p)^2/4\beta\, u \tag{15c}$$

$$\Psi_2 = b(2n + p_s + p)^2/4\beta\, u \tag{15d}$$

$$\Psi_3 = b(2n - p_s - p)^2/4\beta\, u \tag{15e}$$

$$\Psi_4 = b(2n - p_s + p)^2/4\beta\, u \tag{15f}$$

where n denotes the number of images. Then, the analytical solution which includes the side-wall reflection effects can be written as follows:

$$
\begin{aligned}
c = \frac{1}{4Q}\sqrt{\frac{b\,u}{\pi\beta e_x}} \sum_{k=1}^{N} \Bigg[(\Delta W_k)\cdot \\
\Bigg\{ \sum_{n=0}^{\infty} \Bigg[& \frac{\exp\left\{\left(\frac{ux}{2e_x}\right) - 2\sqrt{\eta(\alpha+\Psi_1)}\right\}}{\{\eta(\alpha+\Psi_1)\}^{1/4}} \left\{ \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_1}{T_j}} - \sqrt{\eta T_j}\right) - \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_1}{T_i}} - \sqrt{\eta T_i}\right)\right\} \\
& + \frac{\exp\left\{\left(\frac{ux}{2e_x}\right) - 2\sqrt{\eta(\alpha+\Psi_2)}\right\}}{\{\eta(\alpha+\Psi_2)\}^{1/4}} \left\{ \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_2}{T_j}} - \sqrt{\eta T_j}\right) - \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_2}{T_i}} - \sqrt{\eta T_i}\right)\right\}\Bigg] \\
+ \sum_{n=1}^{\infty} \Bigg[& \frac{\exp\left\{\left(\frac{ux}{2e_x}\right) - 2\sqrt{\eta(\alpha+\Psi_3)}\right\}}{\{\eta(\alpha+\Psi_3)\}^{1/4}} \left\{ \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_3}{T_j}} - \sqrt{\eta T_j}\right) - \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_3}{T_i}} - \sqrt{\eta T_i}\right)\right\} \\
& + \frac{\exp\left\{\left(\frac{ux}{2e_x}\right) - 2\sqrt{\eta(\alpha+\Psi_4)}\right\}}{\{\eta(\alpha+\Psi_4)\}^{1/4}} \left\{ \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_4}{T_j}} - \sqrt{\eta T_j}\right) - \mathrm{erf}\left(\sqrt{\frac{\alpha+\Psi_4}{T_i}} - \sqrt{\eta T_i}\right)\right\}\Bigg]\Bigg\}\Bigg] .
\end{aligned}
\tag{16}
$$

In solving Equation 16, T_i or T_j could be zero or negative in some computational steps depending on the values of t, t_i and t_j under consideration. When T_i (or T_j) is zero, the argument of the error function becomes infinity; and hence, the value of the corresponding error function will be $[\mathrm{erf}(\infty)] = 1.0$. However, when T_i and/or T_j attain negative values, the arguments of the error functions become imaginary; for such computational steps, the concentration values cannot be evaluated. This is consistent with the fact that the inputs at the times $t_i < t$ and $t_j < t$ do not contribute to the concentration distribution, c (x, q, t).

There could be an upstream movement of discharged material when the diffusive transport is greater than the longitudinal advective transport. For such conditions, Equations 8, 14 and 16 are applicable to negative as well as positive values of the longitudinal distance (x) with respect to the origin at the source. The distance affected by the upstream movement (i.e., negative values of x) is dependent on the values of e_y and u.

2.1.4 Times of Passage:

A knowledge of the expected time of arrival and duration of passage of cloud at a given longitudinal distance (x) from the source is necessary to predict concentrations using Equation 16. These can be estimated through the use of the relationships applicable to an instantaneous source in conjunction with the known duration of the finite-time source. The passage times of cloud in the 2-D instantaneous release case can be determined from the following relationship (see Section A.2 in Appendix A):

$$t = \frac{1}{u^2}\left\{ ux + 2e_x \ln (1/\Psi') \pm 2 \sqrt{[\,u\,x\,e_x \ln (1/\Psi')] + [\,e_x \ln (1/\Psi')\,]^2 - [\,e_x\,ub\,(q_s - q)^2/4\beta Q^2\,]} \right\} \qquad (17)$$

where Ψ' is a specified relative concentration. It denotes the approximate ratio of the desired minimum concentration to the peak concentration at a given location. The selection of its value is dependent on the desired accuracy, the general range being 0.1 - 0.01.

Equation 17 gives the passage times as a function of the cumulative discharge (q) at desired transects below the source. For computational convenience, the times obtained with $q_s = q$ (i.e., the maximum passage times) at a given location, will be considered for all values of q. It should be noted that Equation 17 obtained for the 2-D case will be the same as that for the 1-D case when $q = q_s$ (see Heathcote, 1982).

From Equation 17, two values (t_a and t_b) are obtained with the -ve and +ve signs, respectively. The value for t_a corresponds to the arrival time; and the difference ($t_b - t_a$) gives the duration of passage of the cloud for the instantaneous case. The expected arrival time of cloud for the finite-time case will also be equal to t_a. However, the duration of passage (t_p) should be modified to account for the duration of the finite-time input; thus, the resulting expression is

$$t_p = t_b - t_a + t_f \tag{18}$$

in which t_f = duration of input from the finite-time source. The procedures utilized in computer program development to determine the number of time steps and the times for predicting the concentrations at a given transect are described in detail in Section 2.2.

2.1.5 Reach-Dependency of Parameters:

In natural streams and rivers, the channel widths, depths and velocities, longitudinal dispersion coefficients, lateral diffusion factors and pollutant decay rates tend to vary in the longitudinal direction. In such cases, moving average values of the parameters are utilized in the computations (Gowda, 1980; 1984b). The pertinent relationships are presented below.

The moving average values of width, B_i, depth, H_i, and velocity, U_i, applicable to the channel length, x_i, are determined from

$$B_i = \frac{1}{2}\,\frac{\Sigma(b_{i-1} + b_i)(x_i - x_{i-1})}{x_i} \tag{19a}$$

$$H_i = \frac{1}{4}\,\frac{\Sigma(h_{i-1} + h_i)(b_{i-1} + b_i)(x_i - x_{i-1})}{B_i x_i} \tag{19b}$$

$$U_i = Q/B_i H_i \tag{19c}$$

The relationships for moving average values of longitudinal dispersion coefficient (e_{xi}) and transverse diffusion factor (β_i) are

$$e_{xi} = \frac{1}{x_i} \sum_{j=1}^{i} \left\{ e'_{xj} \quad (x_j - x_{j-1}) \right\} \tag{20}$$

$$\beta_i = \frac{1}{x_i} \sum_{j=1}^{i} \left\{ \beta'_j \, (x_j - x_{j-1}) \right\} \tag{21}$$

where e'_{xj} and β'_j denote the values for the jth reach of length $(x_j - x_{j-1})$.

The moving average values of the decay rate coefficients (K_i) are obtained from

$$K_i = \Sigma \, (K'_j \, T_{tj})/T_{ti} \tag{22}$$

where K'_j is the decay rate in the jth reach; T_{tj} is the travel time in the jth reach of length $(x_j - x_{j-1})$ and T_{ti} is the total travel time in the length x_i, extending from the source to the transect, i.

2.1.6 Estimation of the Number of Images:

An examination of Equation 16 indicates that the various summation terms involving the images can be terminated by setting:

$$\exp\left\{ \left(\frac{ux}{2e_x} \right) - 2 \sqrt{\eta(\alpha + \Psi_m)} \right\} = \delta \; ; \quad m = 1 \text{ to } 4 \tag{23}$$

where δ is a positive number to be selected such that the value of each nth term is small enough to cause a negligible effect on c(x, q, t). This procedure is similar to the one utilized in mixing zone models (Gowda, 1980). Taking logarithms

on both sides to base e and simplifying, one obtains quadratic equations of n. The number of images for the four terms involving ψ_m (m = 1 to 4) are then given by the following solutions of the quadratic equations:

$$\Psi_1: \quad n = 0.5 \,(p - p_s) \pm \lambda_1 \tag{24a}$$

$$\Psi_2: \quad n = -0.5 \,(p + p_s) \pm \lambda_2 \tag{24b}$$

$$\Psi_3: \quad n = 0.5 \,(p + p_s) \pm \lambda_3 \tag{24c}$$

$$\Psi_4: \quad n = -0.5 \,(p - p_s) \pm \lambda_4 \tag{24d}$$

where $$\lambda_m = \left[\frac{\beta u}{4b} \left\{ \frac{[\,(ux/2e_x) - \ln (1/\delta)\,]^2}{\eta} - 4\alpha \right\}\right]^{1/2} \tag{24e}$$

subject to the condition that $$[\,(ux/2e_x) - \ln (1/\delta)\,]^2 \geqq 4\eta\alpha \tag{24f}$$

i.e. $$\delta \leqq \exp \left[\left(\frac{ux}{2e_x} \right) - \sqrt{4\eta\alpha} \right] \tag{24g}$$

Generally, the values of δ vary approximately between 1.0 and 10^{-5} for small and large values of x, respectively.
It is recommended to determine the value of δ for a given set of conditions as outlined elsewhere (Gowda, 1980).

From Equations 24 a - d, two values of n are obtained for each case: A lower bound n_1 with the negative sign and a higher bound n_2 with the positive sign.

The total number of images, n_t, for each case is given by the sum of the absolute values of n_1 and n_2. The summations, $n = 0$ to ∞ and $n = 1$ to ∞, appearing in Equation 16, are then caried out from $n = 0$ to n_t and $n = 1$ to n_t, respectively. Generally, the maximum number of images required is nine (Gowda, 1980).

2.2 Computer Program Development

2.2.1 General Description

A computer program for the TWO-DImensional FINite-time model with the acronym TWODIFIN has been developed to perform the computations according to Equations 16-24. A listing of the program, written in FORTRAN language, is presented in Appendix B. A list of the input/output variables and an example showing the input and output of the program are also presented in the appendix. The salient features of the computer program TWODIFIN are presented below.

The inputs to the program include streamflow rate and the corresponding width, depth and velocity, as well as nondimensional transverse diffusion factor and pollutant decay rate for each reach of the river segment under consideration, and the longitudinal distances from the source (i.e., outfall) to each transect. The design temperature and streamflow values are also specified. The widths, depths and velocities for each reach are scaled up to the design streamflow rate using the Loepold-Maddock equations, while the decay rates are adjusted to the design temperature according to the modified van't Hoff-Arrhenius relationship (Equation 5). Then, the moving average values of these parameters are computed.

The longitudinal dispersion coefficients (e_x) for each reach can be directly input to the program by specifying the flag parameter, FLAGEX = 1. Alternatively, if FLAGEX = 2, the e_x values are computed from (Bansal, 1971; Gore & Storrie Ltd., 1983)

$$e_x = [(7.05)(10^6)\ \bar{u}\ \bar{h}\ /\ R_n^{0.762}]\ \mathrm{m^2/s} \qquad (25)$$

where $R_n = (\bar{u}\ \bar{h}/\nu)$ is the Reynolds number, ν being the kinematic viscosity of water (m^2/s) at the known temperature ($T_1{}^\circ$ C).

The viscosity at a design temperature ($T_2°$ C) is calculated from (Rich, 1973)

$$\nu_{T2} = \nu_{T1}\ (1.029)^{T2\ -\ T1} \qquad (26)$$

The discharges, pollutant concentrations and associated times at which the inputs enter the river channel, as well as the location of pipe outfall in the river channel (as a fraction of the river discharge), are also included in the input data. Other data inputs to the program are stated in Appendix B.

The duration of passage (t_p), estimated from Equations 17 and 18 as outlined earlier, is utilized to determine the number of time-steps (n_t) as follows:

If $t_p \leqq 10$, $n_t = 20$
If $10 < t_p \leqq 40$, $n_t = 40$
If $t_p > 40$, $n_t = 50$

The numbers of time steps for the rising and falling portions of the concentration-time curves (n_{t1} and n_{t2}, respectively), are calculated from the empirical relationships $n_{t1} = (0.4\ n_t)$ and $n_{t2} = (0.6\ n_t)$. This empirical procedure is designed to permit concentration predictions at relatively closer intervals on the rising portion of the curve compared with the falling portion which tends to display an elongated tail.

The approximate time (t_r) to the concentration peak at a transect distant (x) from the outfall is calculated from

$$t_r = [(0.8\ x/U) - t_a] \text{ hours} \qquad (27)$$

where U = velocity (m/hr) and t_a = arrival time (hour).

Then, the time-intervals for the rising and falling portions of the curve (Δt_1 and Δt_2, respectively), are obtained from

$$\Delta t_1 = t_r/(n_{t1} - 1), \qquad (28a)$$

$$\text{and} \quad \Delta t_2 = (t_p - t_r)/(n_{t2} - 1) \tag{28b}$$

The times for predicting the concentrations are then determined from the expressions

$$t = [t_a + \Delta t_1 (k - 1)], \quad \text{for } 1 \leqq k \leqq n_{t1}, \tag{29}$$

$$t = [t_r + \Delta t_2 (k - n_{t1})], \quad \text{for } n_{t1} < k \leqq n_{t2}. \tag{30}$$

The output of the TWODIFIN program includes the distance from the source to the transect under consideration, the associated moving average values of the process parameters and the lateral concentration distributions at various passage times at the transect.

It should be noted that the algorithms of TWODIFIN are valid for positive values of x, although the various analytical solutions (i.e., Equations 8, 14 and 16) are applicable to negative values of x as well.

2.2.2 Example

In order to illustrate the data inputs for the TWODIFIN model and the resulting output, an example based on the data gathered following the tritium spill from the Rolphton NPD during August 17 - 25, 1981, is considered. The input data for this example and the computational results are presented in Appendix B. The salient aspects of this illustrative example are discussed below.

The number of transects considered is five (including x = 0). These include the locations at Rolphton, Deep River, Chalk River, Petawawa and Pembroke (see Figure 1 in Chapter III). The channel hydraulic data are known at a flowrate of 298.7 m^3/s. For the tritium spill conditions of August 1981, the streamflow rate is 493.1 m^3/s. The exponents for scale-up of widths, depths and velocities are taken equal to 0.375, 0.375 and 0.25, respectively, for each reach. The longitudinal dispersion coefficients are computed internally in the program, using Equation 25. The value of nondimensional diffusion factor for each reach is assumed to be 0.001. The decay rate of tritium is taken to be zero. The relative concentration (Ψ') is set at 0.1. The flow rate and associated tritium

concentration values at various times were obtained from the concentration versus time plot monitored at the Rolphton NPD (see Figure 2 in Chapter III).

The sequence of input data for the program is described in Section B.3, Appendix B. The input data for this illustrative example are given in Section B.4 of the appendix.

The locations at which the tritium concentrations are computed by the TWODIFIN model include Deep River, Chalk River, Petawawa and Pembroke. The output tables for these four locations are presented in Section B.4, Appendix B. These tables show lateral concentration distributions at q = 0 to 1 (with an interval of 0.2) for various times. The moving average values of the process parameters are also included in the tables.

III ANALYSES RELATED TO CONVECTIVE PERIOD AND LINKING OF NEAR-FIELD AND FAR-FIELD MODELS

3.1 Assessment of the Importance of Convective Period

The theoretical distance downstream of the Ralphton NPD discharge within which convective period is likely to be dominant, has been estimated to be 53.75 km (Gore & Storrie, Ltd., 1983). This estimate was obtained from a relationship developed by Fischer (1967) based on laboratory flume studies. The convective period is characterized by highly skewed concentration-versus-time distributions. McQuivey and Keefer (1976) have developed a model termed "convective model of longitudinal dispersion" which can be utilized to predict concentration distributions in the convective period zone. However, the need for applying the convective model should be determined with the aid of measured concentration-versus-time distributions in the study segment of the river channel. This is necessary since the length within which the convective period is dominant in natural rivers could differ from that predicted by Fischer's equation depending on the presence of meanders, riffles and other conditions that enhance the mixing processes.

In order to assess the importance of the convective period in the Ottawa River below the Rolphton NPD discharge, data collected as a result of an accidental release of tritium-contaminated water from the Rolphton NPD plant during August 1981, will be utilized. The discharge involved a total of 3786 Ci (1 Ci = 1 curie = 27×10^{10} disintegrations per second) over a duration of about nine days (August 17-25, 1981). During the ensuing period, tritium was monitored at the municipal water supply intakes located at Deep River, Petawawa and Pembroke, as well as at Ottawa. Figure 1 shows the study area including the Rolphton NPD plant and the monitoring locations.

The amount of tritium discharged from the Rolphton NPD plant varied with time as shown in Figure 2. Plots of tritium concentration versus time, measured at the three water intake locations and at Ottawa, have been presented in Figure 3. (Note: In Figure 3, the tritium concentrations are in Becquerels/L where 1 Becquerel/L = 27×10^{-12} Ci/L). This figure, prepared by the Radiation Protection Bureau, Health and Welfare Canada (1983), shows the concentration-versus-time plots as step-functions. In order to aid in the identification of any skewnesses in the distributions, smooth curves (shown by broken lines) have been drawn on the plots in Figure 3. An examination of these

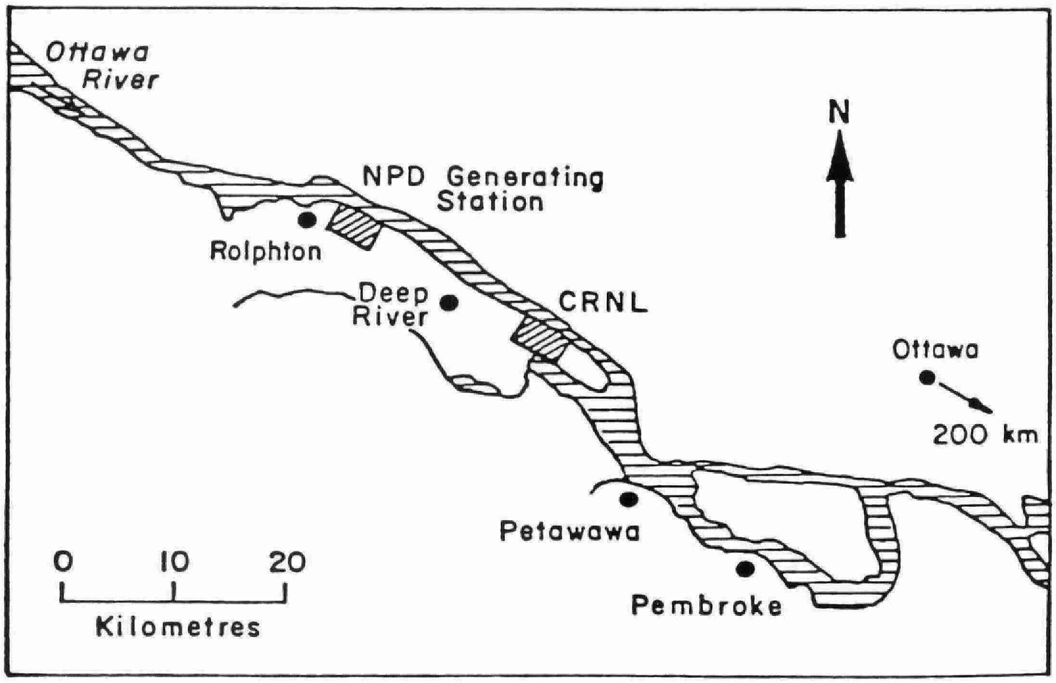

Figure 1. Sampling Locations on the Ottawa River

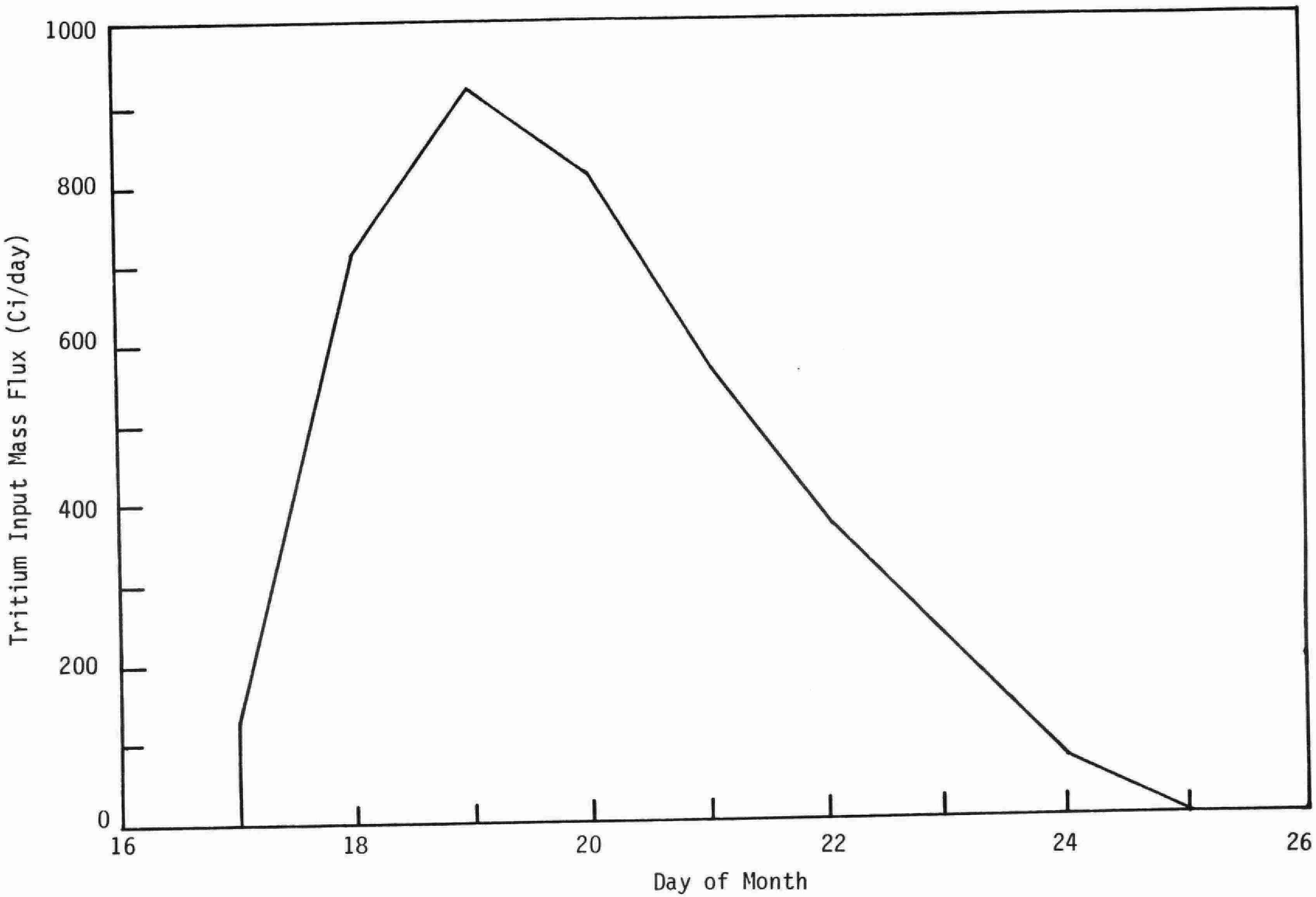

Figure 2. Tritium Spill from Rolphton NPD during August 17-25, 1981

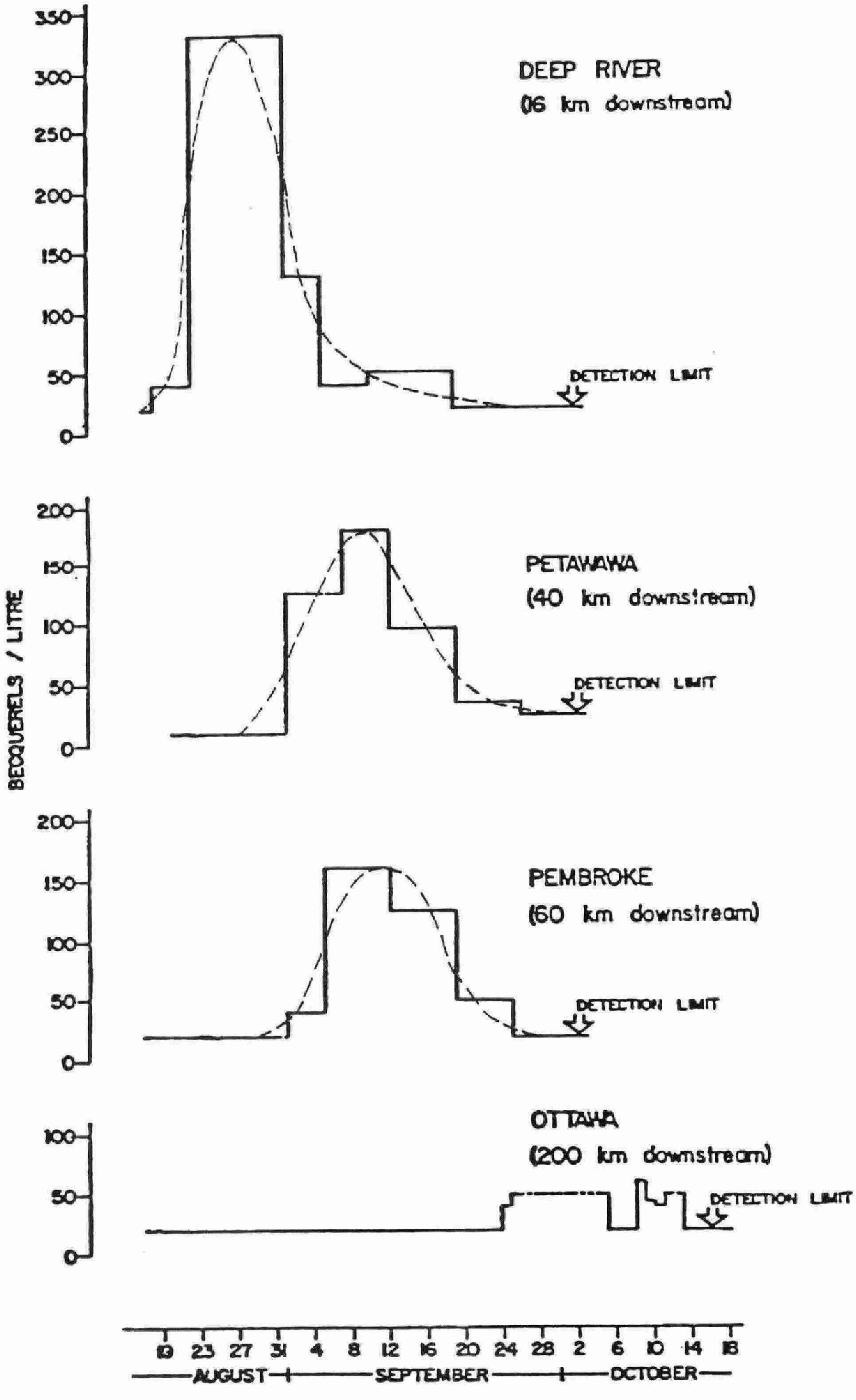

Figure 3. Tritium Concentrations at Sampling Stations on the Ottawa River, August - October, 1981 (Detection limit is 20 Bq/L = 540 pCi/L)

smoothed curves indicates the presence of some degree of skewness in the tritium distribution at the first monitoring site at Deep River (located about 16 km below the Rolphton NPD). The distributions at Petawawa and Pembroke (located about 40 and 60 km respectively, below the Rolphton NPD) do not exhibit a clearly discernible skewness. Therefore, it is reasonable to conclude that the length of the convective period zone in the Ottawa River below the Rolphton NPD discharge would be in the 16-20 km range.

A further examination of the detailed tritium concentration-versus-time plot at the Deep River monitoring site, presented in Figure 4, indicates the following characteristics:

coefficient of skew = 0.18
kurtosis = 2.01

A comparison of these two parameters with the corresponding values for a Gaussian distribution (skewness = 0.0; kurtosis = 3) indicates that the observed tritium distribution at Deep River can be approximated by a Gaussian distribution for modelling purposes. This will permit the utilization of convection-diffusion models without the need for a convective model. However, it is recommended to further evaluate the implications of this approximation through a comparison of observed and predicted durations and time of passage, and concentration distributions for the Deep River site, using the data collected in the study area.

3.2 Relation Between Convective Period and Lateral Mixing Zones

The longitudinal distance below the Rolphton NPD discharge at which complete lateral mixing takes place has been estimated to be 42.66 km. Transverse concentration distributions of tracers for the study segment of the Ottawa River are not readily available to check this estimated distance. However, it is reasonable to assume that lateral concentration gradients will persist for a considerable distance, although the presence of meanders and other turbulent conditions may result in the distance being less than 42.66 km. Thus, there exists a possibility for an overlapping of the convective period and lateral mixing zones. Unfortunately, analyses for this situation, either theoretically or through field studies, have not been reported in the literature. Fortunately, in the Ottawa River study segment, the convective period zone is not likely to have a significant

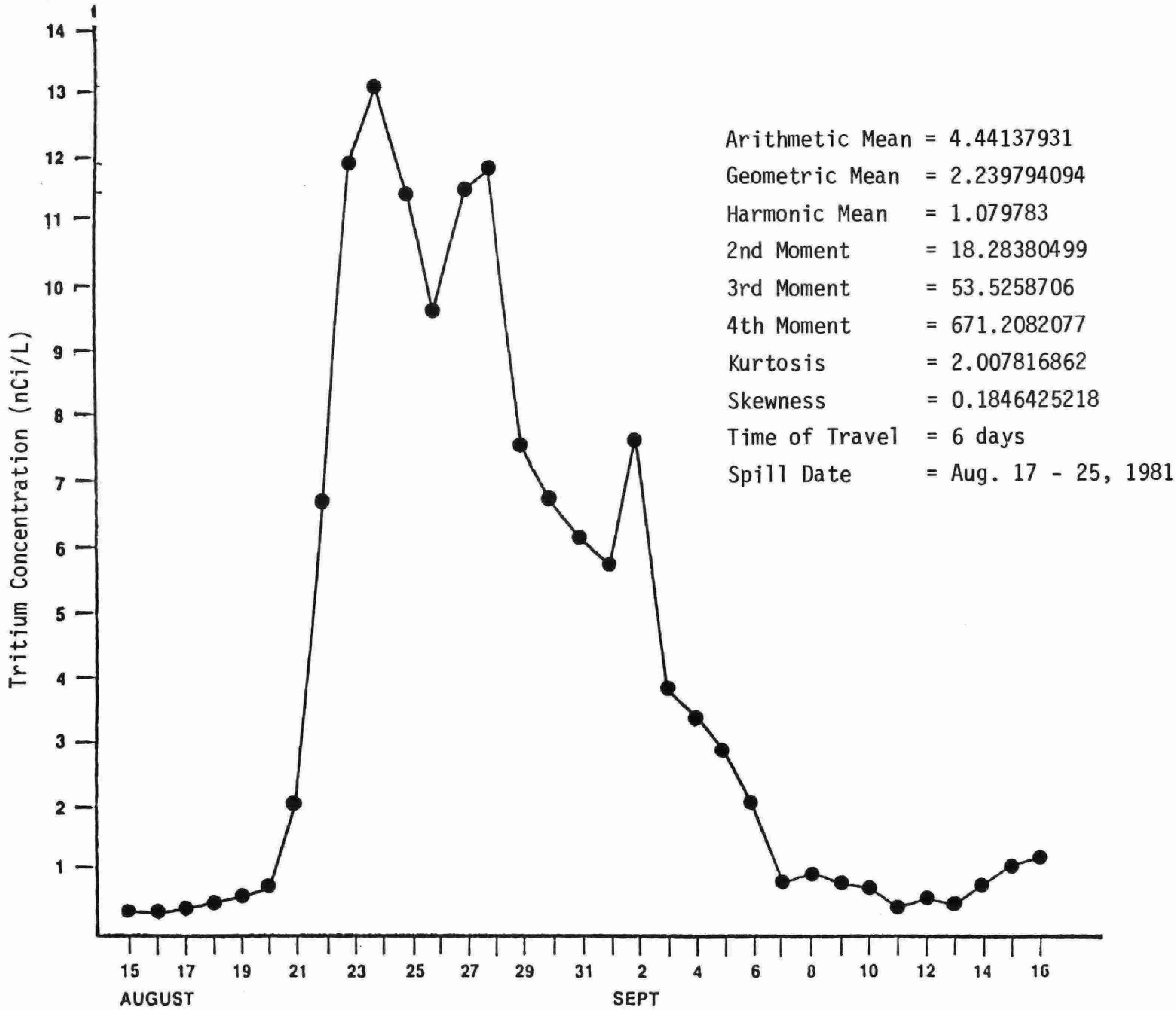

Figure 4. Tritium Distribution at the Deep River Water Supply Intake

effect on the application of convection-diffusion models, as per the analyses presented in the previous section.

It is recommended to further evaluate this aspect with the aid of 2-D model and the data on tracer distributions gathered during the September 1984 field studies in the Ottawa River study segment. If the convective period is dominant within the lateral mixing zone, then the measured concentration distributions would exhibit marked skewness. Based on such an evaluation, the need for further analyses could be determined.

3.3 Linkage Between Near-Field and Far-Field Models

3.3.1 Near-Field and Far-Field Regions:

In water quality modelling studies related to mixing zones, it is a common practice to consider the area of impact of receiving waters as near-field and far-field regions. Each of these regions is characterized by a distinct mixing process in the receiving water. Figure 5 shows a schematic representation of the various zones within these two regions below a bank outfall in a river channel, as well as the type of model applicable to each zone.

The near-field region, closest to actual outfall, is predominantly influenced by the outfall configuration, jet-effect (i.e. momentum of the discharge), as well as bouyancy effects caused by density differences between the effluent discharge and receiving water.

The far-field region, located some distance away from the outfall, is characterized by passive convection and diffusion mechanisms due to ambient flow regime, as well as decay of pollutants. Within the far-field region, concentration gradients may be present in the vertical, lateral and longitudinal directions, the extent of each being dependent on the convection, diffusion and other hydraulic processes in each direction. Generally, the longitudinal distances required to achieve complete vertical and lateral mixing are in the range 50-100 times depth and 50-100 times width of channel, respectively; for the Ottawa River segment below the Rolphton NPD discharge, the corresponding distances are estimated to be 0.345 and 42.66 km (Gore & Storrie Ltd., 1983). Since the zone of vertical mixing in river channels tends to be relatively small (in comparison to the lateral mixing zone), the

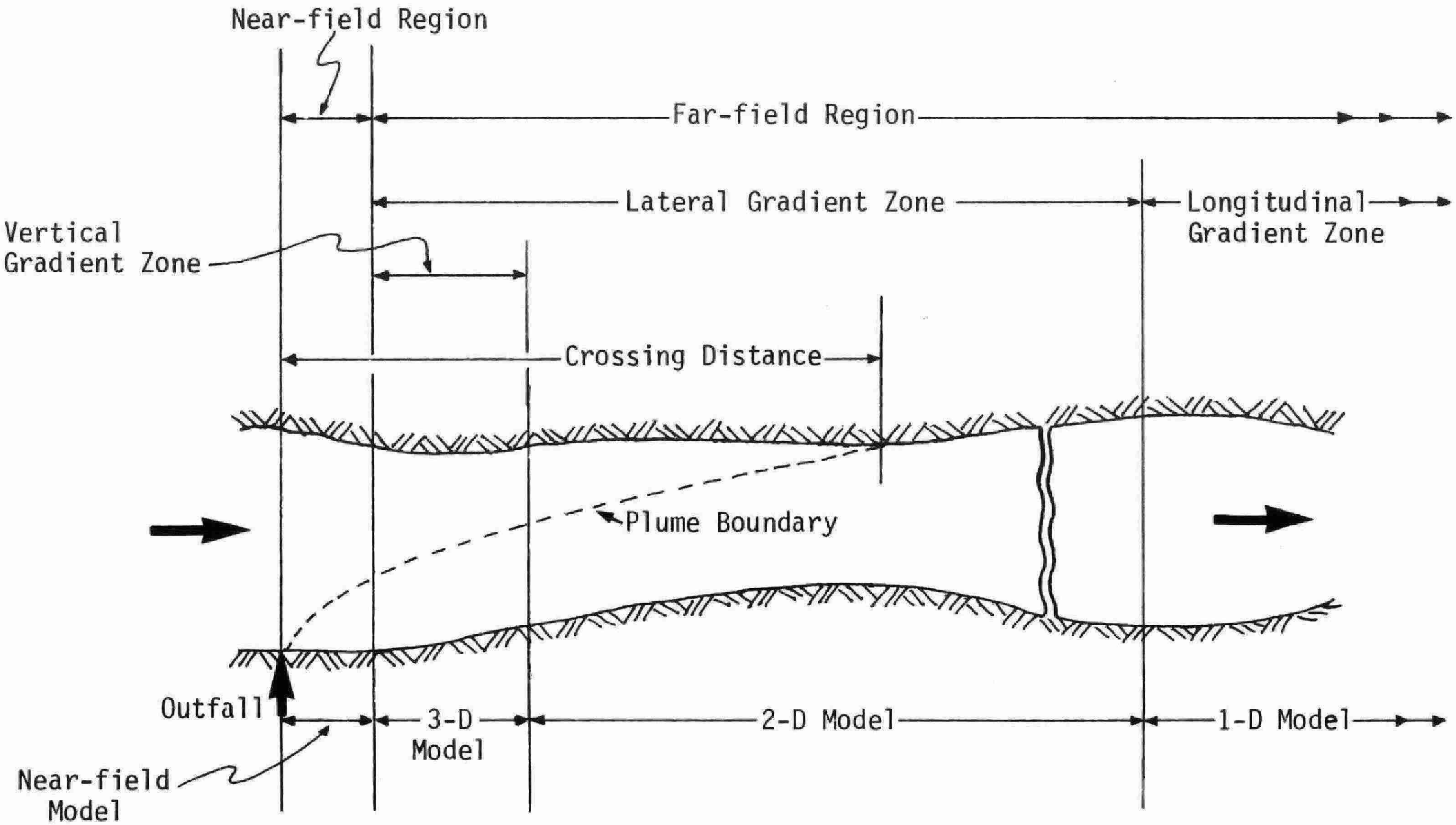

FIGURE 5. SCHEMATIZATION OF NEAR-FIELD AND FAR-FIELD ZONES

linking of near-field and 3-D models is not considered in this study.

3.3.2 Linkage Between Near-Field and Far-Field 2-D Models:

In order to develop a procedure for linking the models applicable to near-field and 2-D transverse mixing zone models, the near-field model developed by Ditmars (1969) and the far-field 2-D models, MIXCALBN and MIXCADIF, have been considered as examples. The theoretical aspects including mathematical formulations of the near-field model can be found in various publications (Hamdy, 1981; Ditmars, 1969). The far-field models are based on the mathematical formulations presented in various publications (Gowda, 1980; Yotsukura and Cobb, 1972); these models are operational on the MOE computer system.

From the predictions of the near-field model, concentration distributions at various distances downstream of the outfall can be obtained. These concentration profiles generally resemble the Gaussian distribution. The nearest distance from the outfall (x_N) at which the near-field model predictions exhibit vertical uniformity can be determined from these predictions.

The far-field 2-D models can then be utilized to predict the concentration distributions at distances equal to or greater than x_N. For the far-field models, the nondimensional transverse diffusion factor (β) applicable to the distance (x_N) should be determined by a model calibration procedure so that the lateral concentration distributions at the distance (x_N) predicted by the near-field and far-field models are in reasonable agreement. This empirical procedure will ensure a continuity in predicting concentration distributions in the near-field and far-field zones.

3.3.3 Linkage Between Far-Field 2-D and 1-D Models

The far-field 2-D model is applicable to the zone within which transverse concentration gradients are predominant. Thus, when the lateral concentration disbributions approach uniformity, the 1-D far-field model can be applied. Theoretically, it is possible to estimate the longitudinal extent of the transverse mixing zone below an effluent discharge (Gowda, 1980 and 1984); for example, the theoretical distance in the Ottawa River channel below the Rolphton NPD discharge is estimated to be 42.66 km, as stated earlier. However, because of the presence

of meanders, etc., the distance could be smaller than 42.66 km. Therefore, it is better to check the predicted lateral concentration distributions at various distances for the attainment of uniformity, and to determine the nearest distance (x_T) below the outfall at which the uniformity of concentrations is achieved. From a practical perspective, the distributions can be considered to be nearly uniform when the absolute percentage difference between the minimum and maximum concentration levels reaches an arbitrary limit of 10 to 20% as suggested by Holley et al (1972). Such a procedure does not significantly affect the predictions and will result in lesser computational time requirements.

The procedure for linking the far-field 2-D and 1-D models outlined above can be easily incorporated for instantaneous, finite-time as well as continuous discharge cases.

IV SUMMARY AND CONCLUSIONS

As part of the Ottawa River nuclear spill contingency model development studies, an analytical solution for the 2-D finite-time case has been derived. The derivation is based on the stream tube concept wherein the lateral co-ordinate is represented by the cumulative discharge. An approximate integration procedure, known as the Laplace method, has been utilized in the derivations. The analytical solution includes the effects of side-wall reflection of the material from channel banks.

Expressions have been obtained for the expected arrival time and duration of passage of spilled material cloud at a given station distant (x) from the source. Expressions are also presented for computing moving average values of reach-dependent parameters which include channel widths, depths and velocities, longitudinal dispersion coefficients, lateral diffusion factors and pollutant decay rates.

A computer pogram for the 2-D finite-time model, TWODIFIN, has been developed in FORTRAN language. The documentation for the program presented in Appendix B includes the program listing, input/output variables and a sample input and ouput.

The importance of the convective period in the Ottawa River below the Rolphton NPD discharge has been assessed with the aid of the tritium concentration-versus-time distributions monitored at Deep River, Petawawa and Pembroke during August - September 1981, following the tritium spill from the Rolphton NPD in August 1981. The convective period is found to dominate for about 16-20 km below Rolphton as indicated by a skewed concentration distribution at Deep River; however, the degree of skewness relative to that for a normal distribution (skewness = 0) is found to be small. Thus, it is concluded that the effect of convective period can be neglected in the modelling studies. It is recommended that this aspect be further examined with the aid of data from the field studies conducted during September 1984 and modelling studies.

The relation between the convective period and lateral mixing zones has been examined. A detailed assessment could not be made because of a lack of relevant information (either theoretical or field study data); it is recommended to further evaluate this aspect with the aid of 2-D model and the field data gathered during the September 1984 field studies.

Methods for linking near-field and far-field models have been presented. A schematic representation shows the various zones in the near-field and far-field regions, and the models applicable to these zones. The methods for linking of models presented herein will permit a continuity in applying the models for the near-field and far-field regions.

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APPENDIX A

MATHEMATICAL DERIVATIONS

A.1 Evaluation of the Integral By The Laplace Method

A.2 Derivation of Expression For Time of Passage

A.1 Evaluation of Integral by the Laplace Method

The integral appearing in Equation 13 is

$$I = \int_{t_i}^{t_j} \exp\left[-\left(\frac{\zeta}{t - t'}\right) - \eta\,(t - t')\right] \frac{dt'}{(t - t')} \qquad \text{(A.1)}$$

Let $(t - t')/t = T$ (A.2)

Then, $dt' = -t.dT; \quad t - t' = tT$ (A.3)

The integration limits will be as follows:

As $t' \longrightarrow t_i, \quad T_i = (t - t_i)/t$ (A.4a)

As $t' \longrightarrow t_j, \quad T_j = (t - t_j)/t$ (A.4b)

With this change of variables, Equation A.1 becomes

$$I = \int_{T_i}^{T_j} \exp\left(-\frac{\zeta}{tT} - \eta t T\right) \frac{dT}{T} \qquad \text{(A.5)}$$

which can be rewritten in the form

$$I = (-1) \int_{T_i}^{T_j} \exp\left[\lambda.h(T)\right] \frac{dT}{T} \qquad \text{(A.6)}$$

where $\lambda = \zeta/t$; (A.7a)

$$h(T) = -\left(\frac{1}{T} + \frac{T}{\alpha^2}\right) \qquad \text{(A.7b)}$$

$$\alpha^2 = (\zeta/\eta t^2) \qquad \text{(A.7c)}$$

For large values of λ, the major contribution to the value of the integral I arises from the immediate vicinity of those points of the integration interval (T_i, T_j) at which h(T) has the largest values. The points at which h(T) attains maxima are obtained by differentiating Equation (A.7b) and setting the first derivative to zero as follows:

$$h'(T) = \frac{d}{dT}[h(T)] = 0 \qquad (A.8a)$$

$$= -\left(\frac{-1}{T^2} + \frac{1}{\alpha^2}\right) = 0 \qquad (A.8b)$$

Thus, we get $T = \alpha$ (A.9)

For h(T) to attain a maximum value, the second derivative of Equation (A.7b) should be negative. This, in fact, is the case as shown below:

$$h''(T) = \frac{d^2}{dT^2}[h(T)] = \frac{-2}{T^3} \qquad (A.10)$$

Because of the fact that the major contribution to the value of the integral, I, is in the vicinity of $T = \alpha$, Equation A.6 can be written in the approximate form

$$I = (-1)\int_{\alpha-\varepsilon}^{\alpha+\varepsilon} \exp\left[\lambda.h(T)\right] \frac{dT}{T} \qquad (A.11)$$

in which ε is some small positive number.

Now, let us define

$$s^2 = h(\alpha) - h(T) \qquad (A.12a)$$

Then $dT = \frac{-2s.ds}{h'(T)}$ (A.12b)

The integration limits are as follows:

$$\text{Lower limit: } s_i = [h(\alpha) - h(\alpha-\varepsilon)]^{1/2} \tag{A.13a}$$

$$\text{Upper limit: } s_j = [h(\alpha) - h(\alpha+\varepsilon)]^{1/2} \tag{A.13b}$$

Note that both s_i and s_j are positive since $h(\alpha)$ is the maximum value of $h(T)$. Substitution of Equations A.12 and A.13 in Equation A.11 results in the following expression for the integral:

$$I = \int_{s_i}^{s_j} \exp\,[\lambda\{h(\alpha) - s^2\}]\ \frac{2.s.ds}{T.h'(T)} \tag{A.14}$$

From Equations A.9 and A.10,

$$\underset{T\to\infty}{\mathrm{Lt}}\left(\frac{1}{T}\right) = \frac{1}{\alpha} \tag{A.15a}$$

$$\underset{T\to\alpha}{\mathrm{Lt}}\left[\frac{s}{h'(T)}\right] = -\ [-2h''(\alpha)]^{-1/2} = -\sqrt{\alpha^3}/2 \tag{A.15b}$$

From Equations A.14 and A.15, we get

$$I = -\sqrt{\alpha}\ \exp\,[\lambda.h(\alpha)] \int_{s_i}^{s_j} \exp\,(-\lambda s^2)\ ds \tag{A.16}$$

From Equations A.7b and A.9,

$$h(\alpha) = -\ (2/\alpha) \tag{A.17}$$

Now, let us express the integration limits s_i and s_j in terms of T_i and T_j. Substitution of Equations A.7b and A.17 in Equation A.12a results in

$$s^2 = [h(\alpha) - h(T)] = \left(\frac{-2}{\alpha} + \frac{1}{T} + \frac{T}{\alpha^2}\right) = \frac{(T - \alpha)^2}{T\alpha^2} \qquad (A.18)$$

Then, $$s_i = \frac{T_i - \alpha}{\alpha\sqrt{T_i}} \qquad (A.19a)$$

$$s_j = \frac{T_j - \alpha}{\alpha\sqrt{T_j}} \qquad (A.19b)$$

The integral appearing on the right hand side of Equation A.16 can now be evaluated as follows

$$\int_{s_i}^{s_j} \exp(-\lambda s^2).ds = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}\,[\mathrm{erf}(s\sqrt{\lambda})]\Big|_{s_i}^{s_j} = \sqrt{\pi/4\lambda}\,[\mathrm{erf}(s_j\sqrt{\lambda}) - \mathrm{erf}(s_i\sqrt{\lambda})]$$

$$= \sqrt{\frac{\pi}{4\alpha}}\left\{\mathrm{erf}\left[\left(\frac{T_j - \alpha}{\alpha}\right)\sqrt{\frac{\lambda}{T_j}}\right] - \mathrm{erf}\left[\left(\frac{T_i - \alpha}{\alpha}\right)\sqrt{\frac{\lambda}{T_i}}\right]\right\} \qquad (A.20)$$

Substituting the expressions from Equations A.17 and A.20 in Equation A.16, the following expression for the integral, I, is obtained:

$$I = -\sqrt{\frac{\pi\alpha}{4\lambda}}.\exp(-2\lambda/\alpha)\left\{\mathrm{erf}\left[\left(\frac{T_j - \alpha}{\alpha}\right)\sqrt{\frac{\lambda}{T_j}}\right] - \mathrm{erf}\left[\left(\frac{T_i - \alpha}{\alpha}\right)\sqrt{\frac{\lambda}{T_i}}\right]\right\} \qquad (A.21)$$

From Equations A.4 and A.7,

$$\mathrm{erf}\left[\left(\frac{T_j - \alpha}{\alpha}\right)\sqrt{\frac{\lambda}{T_j}}\right] = -\,\mathrm{erf}\left(\sqrt{\frac{\zeta}{T_j}} - \sqrt{\eta T_j}\right) \qquad (A.22)$$

Similar expressions for the other error function term of Equation A.21 can be obtained. Therefore, the final expression for the integral appearing in Equation 13 becomes:

$$I = \sqrt{\frac{\pi}{4}} \cdot \frac{\exp(-2\sqrt{\zeta\eta})}{(\zeta\eta)^{1/4}} \cdot \left[\mathrm{erf}\left(\sqrt{\frac{\zeta}{T_j}} - \sqrt{\eta T_j}\right) - \mathrm{erf}\left(\sqrt{\frac{\zeta}{T_i}} - \sqrt{\eta T_i}\right)\right] \qquad (A.23)$$

A.2 Derivation of Expression for Time of Passage

Let us consider Equation 8 with $K_d = 0$, and define a relative concentration as

$$\Psi' = c_I/c_{peak} = c_I \left(2\pi t\sqrt{E_x E_q}/W_I\, u\right) \qquad (A.24)$$

in which c_{peak} = peak concentration which occurs along $q = q_s$ at $x = ut$. As stated in Section 2.1.4, Equation A.24 gives an approximate ratio of a desired minimum concentration to the peak concentration at a given location. From Equations 8 and A.24, we get the expression

$$\Psi' = \exp\left[-\frac{(x - ut)^2}{4E_x t} - \frac{(q_s - q)^2}{4E_q t}\right] \qquad (A.25)$$

Taking logarithms to base e and simplifying, we get the following quadratic equation in t:

$$t^2 - [(2x/u) - (4\,E_x/u^2)\ln\Psi']t + \{(x/u)^2 + (E_x/E_q)\,[(q_s - q)/u]^2\} = 0 \qquad (A.26)$$

For simplicity, we consider $m_x = 1$ and $m_y = 1$, and substitute $E_q = (BQ^2u/b)$ from Equation 15a. Then, the solution of Equation A.26 is given by

$$t = \frac{1}{u^2}\left\{ux + 2e_x \ln(1/\Psi') \pm 2\sqrt{[u\,x\,e_x \ln(1/\Psi')] + [e_x \ln(1/\Psi')]^2 - [e_x\,ub\,(q_s - q)^2/4BQ^2]}\right\} \qquad (A.27)$$

Equation A.27 is the same as Equation 17 given in Chapter II.

APPENDIX B

DOCUMENTATION FOR

TWODIFIN MODEL - COMPUTER PROGRAM

B.1 Description of Variable Names

TITLE	Title of study.
NX	Number of transects (including the first transect at the source)
NQ	Number of elemental strips at transect
NY	Number of lateral data points = NQ + 1
IT	Number of input time-intervals
RQS	Cumulative discharge between reference bank and pipe outfall location in the cross-section expressed as a fraction of river discharge below the outfall.
THETA	Temperature correction factor associated with the decay rate (K).
DLT	Value of δ for estimating the number of images.
PSIP	Value of Ψ' for estimating passage times of cloud.
TE	Input times from source or outfall (IT values), hours.
QE	Input discharges from the outfall (IT values), m^3/s.
CE	Input concentrations (IT values).
X	Distance from outfall to a given transect (NX values including a transect at X = 0), meters.
EXD	Longitudinal dispersion coefficients (NX values) for successive reaches of lengths $(x_j - x_{j-1})$, m^2/s
BETA	Non-dimensional transverse diffusion factor values for the successive reaches (NX values).
BP	Width exponent of the Leopold-Maddock equation for each reach.
ZP	Depth exponent of the Leopold-Maddock equation for each reach.
UP	Velocity exponent of the Leopold-Maddock equation for each reach.
AS(I,1)	Reference temperatures (°C) in successive reaches at which the decay rates are known.
AS(I,2)	Reference streamflow rates in successive reaches, m^3/s
AS(I,3)	Reference channel widths in successive reaches, meters.
AS(I,4)	Reference channel depths in successive reaches, meters.
AS(I,5)	Reference channel velocities in successive reaches, m/s.

AS(I,6)	Reference decay rates (base e) in successive reaches, per second.
AD(I,1)	Design temperatures ($^{\circ}C$) in successive reaches.
AD(I,2)	Design streamflow rates in successive reaches, m^3/s
AD(I,3)	Channel width at the design streamflow, meters.
AD(I,4)	Channel depth at the design streamflow, streamflow.
AD(I,5)	Channel velocity at the design streamflow, m/s.
AD(I,6)	Decay rate coefficients at the design temperatures, per second.
B	Moving average value of channel width for a channel length x_i (from outfall to transect, i), meters.
Z	Moving average value of channel depth, meters.
U	Moving average value of channel velocity, m/s.
RK	Moving average value of decay rate coefficient, per second.
EXX	Moving ave. value of longitudinal dispersion coefficient, m^2/s.
BT	Moving average value of the transverse diffusion factor.
TM1	Time of arrival of cloud (t_a), hours.
TM2	Departure time of cloud (t_b), hours.
TMR	Duration of cloud passage (t_p), hours.
QY	Partial cumulative discharge (q), m^3/s.
QT	River discharge, m^3/s
RQ	Ratio (QY/QT).
WT	Mass flux input during a specified time-interval.
WIN	Total mass flux input from the source.
TM	Time of concentration prediction (t), hours and days.
CT	Concentration at a point (x,q) at a time, t.
VISC	Kinematic viscosity of water at the reference temperature, m^2/s.
VIST	Kinematic viscosity of water at the design temperature, m^2/s.
FLAGEX	Flag for e_x; 1 for direct input and 2 for calculating e_x from Bansal's relationship.

B.2 List of TWODIFIN Program

```
C  TWODIFIN: 2-D FINITE TIME FAR-FIELD MODEL FOR VERTICAL LINE SOURCE IN RIVERS
C  PROGRAM DEVELOPED BY:  DR. T. P. H. GOWDA, WATER RESOURCES DIVISION,
C                         GORE & STORRIE LTD. , TORONTO, ONTARIO,
C                            FOR THE  WATER RESOUCES BRANCH,
C                         ONTARIO MINISTRY OF THE ENVIRONMENT, TORONTO.
C                                SEPTEMBER 1984
C
C   * * * * * * * * * *   M A I N    P R O G R A M   * * * * * * * * * *
C
      DIMENSION TITLE(20),CT(6,21,51),NTM(6),IM1(21),IM2(21),KTM(21)
      REAL*8 EXX(6),A1,A2,A3,A4,A5,A6,AD(6,7),AS(6,7),SUMT,WT(10),
     1SBETA(6),RQ(21),X(6),RK(6),BP,ZP,UP,QRTO,QSO,PQ,RQS,TIN,TMR,
     2BXI,TM1,TM2,AP1(21,30),AP2(21,30),AP3(21,30),AP4(21,30),TOT,
     3B1(21,30),B2(21,30),B3(21,30),B4(21,30),D1,D2,D3,D4,SMEXD(6),
     4C1,C2,C3,C4,BEG,TI,TJ,BETA(6),TM(6,51),TE(10),EXD(6),AMBD,DLT,
     5BSUM(6),SUMK(6),STOT(6),VOL(6),THETA,STI(51,30),STJ(51,30),XX,
     6PRN(21),PRP(21),QE(10),CE(10),PSP(21),PTP(21),REYNO,TRIS,TFAL,
     7TR1,TR2,DPVL,PSIP,VISC,VIST,BT(6),B(6),Z(6),U(6)
C
C  DATA INPUT
      READ(1,10)TITLE
      READ(1,*) NX,NQ,IT,RQS,THETA,DLT,PSIP,VISC,IFLAG
      DO 20 I=1,IT
   20 READ(1,*)TE(I),QE(I),CE(I)
C  READ HYDRAULIC, DECAY RATE AND TEMP. DATA FOR EACH TRANSECT FOR
C  REFERENCE AND DESIGN CONDITIONS
      DO 22 I=1,NX
      GO TO(30,32),IFLAG
   30 READ(1,*) X(I),EXD(I),BETA(I),BP,ZP,UP
      GO TO 34
   32 READ(1,*) X(I),BETA(I),BP,ZP,UP
   34 READ(1,*) (AS(I,IS),IS=1,6)
      READ(1,*) (AD(I,IS),IS=1,2)
C  SCALE-UP HYDRAULIC PARAMETERS AND DECAY RATES TO DESIGN CONDITION
      PNQ=FLOAT(NQ)
      PQ=DBLE(1.0/PNQ)
      QRTO=AD(I,2)/AS(I,2)
      AD(I,3)=AS(I,3)*QRTO**BP
      AD(I,4)=AS(I,4)*QRTO**ZP
      AD(I,5)=AS(I,5)*QRTO**UP
      AD(I,6)=AS(I,6)*THETA**(AD(I,1)-AS(I,1))
      GO TO(22,36),IFLAG
C  CALCULATE LONG'L DISP'N COEFF. USING BANSAL'S EQN.
   36 DPVL=AD(I,4)*AD(I,5)
      VIST=VISC*1.029**(AS(I,1)-AD(I,1))
      REYNO=DPVL/VIST
      EXD(I)=(7050000.0)*DPVL/REYNO**0.762
   22 CONTINUE
C  CALCULATE INPUT MASS FLUX FOR EACH TIME STEP AND TOTAL INPUT FLUX
      IDT=IT-1
      WIN=0.0
      DO 24 L=1,IDT
      LL=L+1
      TIN=3600.0*(TE(LL)-TE(L))
      WT(L)=0.5*(QE(L)*CE(L)+QE(LL)*CE(LL))
   24 WIN=WIN+WT(L)*TIN
      NY=NQ+1
      DO 38 J=1,NY
```

```
      RQ(J)=PQ*(J-1)
      PRN(J)=RQS-RQ(J)
   38 PRP(J)=RQS+RQ(J)
C  COMPUTE MOVING AVERAGE VALUES OF HYDRAULIC PARAMETERS, LONG'L AND
C  LAT'L DISPERSION COEFFTS., AND DECAY RATE
      SUMK(1)=0.0
      VOL(1)=0.0
      BSUM(1)=0.0
      STOT(1)=0.0
      SMEXD(1)=0.0
      SBETA(1)=0.0
      DO 76 I=2,NX
      I1=I-1
      XX=X(I)-X(I1)
      BXI=XX*(AD(I,3)+AD(I1,3))
      BSUM(I)=BSUM(I1)+0.5*BXI
      VOL(I)=VOL(I1)+0.25*BXI*(AD(I,4)+AD(I1,4))
      TOT=XX/AD(I,5)
      SUMK(I)=SUMK(I1)+(TOT*AD(I,6))
      STOT(I)=STOT(I1)+TOT
      SMEXD(I)=SMEXD(I1)+XX*EXD(I)
      SBETA(I)=SBETA(I1)+XX*BETA(I)
      B(I)=BSUM(I)/X(I)
      Z(I)=VOL(I)/BSUM(I)
      U(I)=AD(I,2)/(B(I)*Z(I))
      RK(I)=SUMK(I)/STOT(I)
      EXX(I)=SMEXD(I)/X(I)
      BT(I)=SBETA(I)/X(I)
      BEQ=B(I)/(BT(I)*U(I))
C  CALCULATE VALUES OF TERMS DEPENDENT ON X(I)
      A1=(1.0/(7.0898154*AD(I,2)))*DSQRT((B(I)*U(I))/(EXX(I)*BT(I)))
      A2=4.0*EXX(I)
      A3=DSQRT((U(I)*U(I)/A2)+RK(I))
      A4=X(I)**2/A2
      A5=U(I)*X(I)/(2.0*EXX(I))
      A6=0.25*(A5+DLOG(1.0/DLT))**2/(A3*A3)-A4
      IF(A6.LT.0.0) A6=0.0
      AMBD=DSQRT(A6/BEQ)
C  INITIALIZE IMAGE TERM ARRAY VALUES TO ZERO
      DO 70 J=1,NY
      IM1(J)=0
      IM2(J)=0
      DO 70 N=1,30
      AP1(J,N)=0.0
      AP2(J,N)=0.0
      AP3(J,N)=0.0
      AP4(J,N)=0.0
      B1(J,N)=0.0
      B2(J,N)=0.0
      B3(J,N)=0.0
   70 B4(J,N)=0.0
C  COMPUTE AND STORE VALUES OF IMAGE TERMS INDEPENDENT OF TIME
      DO 74 J=1,NY
      CALL RIMAGE(AMBD,PRN,J,M1,M4)
      CALL RIMAGE(AMBD,PRP,J,M2,M3)
      IM1(J)=AMAX0(M1,M2)
      IM2(J)=AMAX0(M3,M4)
      MJ1=IM1(J)
      MJ2=IM2(J)
      PSP(J)=-PRN(J)
```

```
      PTP(J)=-PRP(J)
      DO 72 N=1,MJ1
      CALL REFLEC(PRN,BEQ,A3,A4,A5,J,N,AP1,B1)
      CALL REFLEC(PRP,BEQ,A3,A4,A5,J,N,AP2,B2)
   72 CONTINUE
      DO 74 N=2,MJ2
      CALL REFLEC(PTP,BEQ,A3,A4,A5,J,N,AP3,B3)
      CALL REFLEC(PSP,BEQ,A3,A4,A5,J,N,AP4,B4)
   74 CONTINUE
C  GENERATE PASSAGE TIMES TM(I,K) AND COMPUTE VALUES OF TIME-DEPENDENT TERMS
      CALL PASTIM(X,U,EXX,PSIP,I,TM1,TM2)
      TMR=TM2-TM1+TE(IT)-TE(1)
      IF(TMR.LE.10.0) NT=20
      IF(TMR.GT.10.0.AND.TMR.LE.40.0) NT=40
      IF(TMR.GT.40.0) NT=50
      TRIS=(0.8*X(I)/(3600.0*U(I)))-TM1
      TFAL=TMR-TRIS
      TFN=FLOAT(NT)
      TFN=0.4*TFN+0.5
      NT1=IFIX(TFN)
      NT1=0.4*NT
      NT2=NT-NT1
      TR1=TRIS/(NT1-1)
      TR2=TFAL/(NT2-1)
      DO 26 K=1,NT
      IF(K.LE.NT1) TM(I,K)=TM1+TR1*(K-1)
      IF(K.GT.NT1) TM(I,K)=TM(I,NT1)+TR2*(K-NT1)
      DO 26 L=1,IDT
      LL=L+1
      TI=3600.0*(TM(I,K)-TE(L))
      TJ=3600.0*(TM(I,K)-TE(LL))
      IF(TI.LT.0.0.OR.TJ.LT.0.0)GO TO 40
      STI(K,L)=DSQRT(TI)
      STJ(K,L)=DSQRT(TJ)
      GO TO 26
   40 STI(K,L)=-99.9
      STJ(K,L)=-99.9
   26 CONTINUE
C  CALCULATE CONCN'S FOR EACH POINT (I,J) AT EACH TIME TM(I,K)
      DO 98 J=1,NY
      KTM(J)=0
      M1=IM1(J)
      M2=IM2(J)
      DO 96 K=1,NT
      SUMT=0.0
      DO 78 L=1,IDT
      IF(STI(K,L).LT.0.0.OR.STJ(K,L).LT.0.0)GO TO 84
      C1=0.0
      C2=0.0
      C3=0.0
      C4=0.0
      DO 80 N=1,M1
      CALL FUNER(AP1,STI,STJ,A3,J,N,K,L,D1)
      CALL FUNER(AP2,STI,STJ,A3,J,N,K,L,D2)
      C1=C1+B1(J,N)*D1
   80 C2=C2+B2(J,N)*D2
      DO 82 N=2,M2
      CALL FUNER(AP3,STI,STJ,A3,J,N,K,L,D3)
      CALL FUNER(AP4,STI,STJ,A3,J,N,K,L,D4)
      C3=C3+B3(J,N)*D3
```

```
 82   C4=C4+B4(J,N)*D4
      SUMT=SUMT+WT(L)*(C1+C2+C3+C4)
 78   CONTINUE
 84   CT(I,J,K)=A1*SUMT
      IF(K.LE.19.OR.K.EQ.NT)GO TO 86
      IF(CT(I,J,K).GE.0.00001) GO TO 86
      KTM(J)=K
      GO TO 88
 86   CONTINUE
      KTM(J)=NT
 88   CONTINUE
      NTM(I)=-1
      DO 28 J=1,NY
      IF(NTM(I).GT.KTM(J)) GO TO 28
      NTM(I)=KTM(J)
 28   CONTINUE
      NT=NTM(I)
      DO 76 J=1,NY
      KM=KTM(J)+1
      IF(KM.LE.20) GO TO 76
      IF(KM.GE.NT) GO TO 76
      DO 42 K=KM,NT
 42   CT(I,J,K)=0.0
 76   CONTINUE
C  PRINT CONC'N DISTR'N AT X(I,J) AT THE TIMES TM(I,K)
      WRITE(1,10) TITLE
      IY=NY/10
      IF(IY.LE.1) IY=1
      DO 44 I=2,NX
      NT=NTM(I)
      ISTN=I-1
      WRITE(1,52) ISTN
      WRITE(1,12) X(I),AD(I,1),AD(I,2)
      WRITE(1,14) B(I),Z(I),U(I),EXX(I),BT(I),RK(I)
      WRITE(1,16)(RG(J),J=1,NY,IY)
      DO 44 K=1,NT
      TDAY=TM(I,K)/24.0
 44   WRITE(1,18) K,TM(I,K),TDAY,(CT(I,J,K),J=1,NY,IY)
C  FORMAT STATEMENTS
 10   FORMAT(20A4)
 12   FORMAT(3X,'X=',F10.1,3X,'TEMP=',F6.2,3X,'Q=',F8.2)
 14   FORMAT(3X,'AVG. VALUES: ',2X,'B=',F7.2,2X,'Z=',F6.2,2X,'U=',F6.3,
     *3X,'EX=',F8.3,2X,'BETA=',F7.4,2X,'KD=',E10.4/)
 16   FORMAT(2X,'NO. ',1X,'TIME(HRS.:DAYS)',11(1X,F9.2)/)
 18   FORMAT(1X,I3,1X,F8.2,2X,F5.1,1X,11(1X,F9.4))
 52   FORMAT(/3X,'STATION ',I2)
      STOP
      END
C
C   * * * * * *     S U B R O U T I N E S     * * * * * *
C
C  CALCULATE ERROR FUNCTION VALUES
C
      SUBROUTINE FUNER(AP,SI,SJ,AR,K,M,K1,L1,D)
      REAL*8 AP(21,30),SI(51,30),SJ(51,30),AR,D,BI,BJ
      IF(SJ(K1,L1).EQ.0.0)GO TO 10
      BJ=DERF((AP(K,M)/SJ(K1,L1))-AR*SJ(K1,L1))
      GO TO 12
 10   BJ=1.0
 12   IF(SI(K1,L1).EQ.0.0)GO TO 14
```

```
      BI=DERF((AP(K,M)/SI(K1,L1))-AR*SI(K1,L1))
      GO TO 16
   14 BI=1.0
   16 D=BJ-BI
      RETURN
      END
C
C  ESTIMATION OF PASSAGE TIMES OF CLOUD
C
      SUBROUTINE PASTIM(XI,UI,EXI,PSI,IX,TX1,TX2)
      REAL*8 XI(6),UI(6),EXI(6),TX1,TX2,PSX,UX,UT,XUE,SXU,PSI
      PSX=EXI(IX)*DLOG(1.0/PSI)
      UX=UI(IX)*XI(IX)
      UT=UI(IX)*UI(IX)*3600.0
      XUE=UX+2.0*PSX
      SXU=PSX*(UX+PSX)
      SXU=2.0*DSQRT(SXU)
      TX1=(XUE-SXU)/UT
      IF(TX1.LE.0.0) TX1=0.05
      TX2=0.8*(XUE+SXU)/UT
      RETURN
      END
C
C  COMPUTE VALUES OF IMAGE-DEPENDENT TERMS
C
      SUBROUTINE REFLEC(PR,BE,ETA,ALPA,AX,M,K,AN,BN)
      REAL*8 PR(21),AN(21,30),BN(21,30),AR,BE,ETA,ALPA,AX,PN
      PN=(2.0*(K-1)+PR(M))**2
      AN(M,K)=DSQRT(ALPA+0.25*BE*PN)
      AR=ETA*AN(M,K)
      BN(M,K)=DEXP(AX-2.0*AR)/DSQRT(AR)
      RETURN
      END
C
C  CALCULATE THE NUMBER OF IMAGES
C
      SUBROUTINE RIMAGE(AM,PR,J,MT1,MT2)
      REAL*8 AM,PR(21)
      AN1=-0.5*PR(J)-AM-0.5
      AN2=-0.5*PR(J)+AM+0.5
      N1=INT(AN1)
      N2=INT(AN2)
      MT1=IABS(N1)+IABS(N2)
      AN3=0.5*PR(J)-AM-0.5
      AN4=0.5*PR(J)+AM+0.5
      N3=INT(AN3)
      N4=INT(AN4)
      MT2=IABS(N3)+IABS(N4)
      IF(MT1.LE.1) MT1=1
      IF(MT2.LE.2) MT2=2
      IF(MT1.GT.30) MT1=30
      IF(MT2.GT.30) MT2=30
      RETURN
      END
```

B.3 Input Data Sequence

The sequence of data inputs are as follows:

Variable	Format	Remarks
TITLE	20A4	One line
NX, NQ, IT, RQS, THETA, DLT, PSIP, FLAGEX	Free	One line
TE(I), QE(I), CE(I)	Free	IT number of lines
If (FLAGEX = 1): X(I), EXS(I), BETA(I), BP, ZP, UP	Free	Each set of these three lines is entered NX times.
If (FLAGEX = 2): X(I), BETA(I), BP, ZP, UP	Free	
AS(I, IS), IS = 1,6	Free	
AD(I, ID), ID = 1,2	Free	

The program is set up to read the input data from an auxiliary device such as a disk or a tape.

B.4 Example of Input/Output

Input Data

```
TRITIUM SPILL FROM ROLPHTON NPD TO OTTAWA R.: AUG. 17-25, 1981.
 5 5 9 0.0 1.03 0.001 0.1 0.0000010101 2
0. 10. 0.
1.0 10.0 130.0
24. 10. 740.
48. 10. 920.
72. 10. 810.
96.0 10.0 580.0
120.0 10.0 400.0
168. 10. 80.0
192.0 10.0 0.0
 0.0 0.001 0.375 0.375 0.25
20.0 298.7 1170.0 16.21 0.0157 0.0
20.0 493.1
 15788.0 0.001 0.375 0.375 0.25
20.0 298.7 1170.0 16.21 0.0157 0.0
20.0 493.1
 27295.0 0.001 0.375 0.375 0.25
20.0 298.7 1016.0 18.49 0.0159 0.0
20.0 493.1
 45472.0 0.001 0.375 0.375 0.25
20.0 298.7 1800.0 15.65 0.0106 0.0
20.0 493.1
 62292.0 0.001 0.375 0.375 0.25
20.0 298.7 1777.0 4.36 0.0386 0.0
20.0 493.1
```

Output

TRITIUM SPILL FROM ROLPHTON NPD TO OTTAWA R. : AUG. 17-25, 1981.

STATION 1
X= 15788.0 TEMP= 20.00 Q= 493.10
AVG. VALUES: B=1411.96 Z= 19.56 U= 0.018 EX= 148.064 BETA= 0.0010 KD=0.0000E 00

NO.	TIME(HRS.	: DAYS)	0.00	0.20	0.40	0.60	0.80	1.00
1	36.73	1.5	1.3749	0.0014	0.0000	0.0000	0.0000	0.0000
2	45.60	1.9	2.7719	0.0100	0.0000	0.0000	0.0000	0.0000
3	54.48	2.3	5.4843	0.0379	0.0000	0.0000	0.0000	0.0000
4	63.36	2.6	8.7444	0.1013	0.0000	0.0000	0.0000	0.0000
5	72.24	3.0	12.6790	0.2196	0.0000	0.0000	0.0000	0.0000
6	81.12	3.4	17.0205	0.4121	0.0000	0.0000	0.0000	0.0000
7	89.99	3.7	21.5409	0.6941	0.0002	0.0000	0.0000	0.0000
8	98.87	4.1	26.1783	1.0751	0.0005	0.0000	0.0000	0.0000
9	107.75	4.5	30.5418	1.5574	0.0011	0.0000	0.0000	0.0000
10	116.63	4.9	34.4890	2.1366	0.0025	0.0000	0.0000	0.0000
11	125.50	5.2	38.1958	2.8020	0.0048	0.0000	0.0000	0.0000
12	134.38	5.6	41.2804	3.5386	0.0088	0.0000	0.0000	0.0000
13	143.26	6.0	43.7243	4.3285	0.0148	0.0000	0.0000	0.0000
14	152.14	6.3	45.2423	5.1521	0.0236	0.0000	0.0000	0.0000
15	161.02	6.7	45.8284	5.9879	0.0357	0.0000	0.0000	0.0000
16	169.89	7.1	47.8750	6.8238	0.0519	0.0000	0.0000	0.0000
17	178.77	7.4	48.3728	7.6337	0.0727	0.0001	0.0000	0.0000
18	187.65	7.8	48.6028	8.4060	0.0984	0.0001	0.0000	0.0000
19	196.53	8.2	48.5121	9.1312	0.1296	0.0002	0.0000	0.0000
20	240.20	10.0	43.7084	11.8399	0.3636	0.0018	0.0000	0.0000
21	283.86	11.8	36.7844	12.9605	0.7052	0.0078	0.0000	0.0000
22	327.53	13.6	30.7595	12.8959	1.0863	0.0218	0.0000	0.0000
23	371.20	15.5	25.8918	12.2139	1.4376	0.0465	0.0000	0.0000
24	414.87	17.3	21.9793	11.2732	1.7176	0.0815	0.0000	0.0000
25	458.54	19.1	18.8062	10.2607	1.9135	0.1243	0.0000	0.0000
26	502.20	20.9	16.2027	9.2673	2.0305	0.1709	0.0000	0.0000
27	545.87	22.7	14.0425	8.3346	2.0812	0.2175	0.0000	0.0000
28	589.54	24.6	12.2322	7.4787	2.0801	0.2608	0.0000	0.0000
29	633.21	26.4	10.7019	6.7035	2.0404	0.2986	0.0000	0.0000
30	676.87	28.2	9.3983	6.0064	1.9733	0.3299	0.0000	0.0000
31	720.54	30.0	8.2805	5.3821	1.8877	0.3542	0.0000	0.0000
32	764.21	31.8	7.3167	4.8244	1.7906	0.3715	0.0000	0.0000
33	807.88	33.7	6.4814	4.3266	1.6871	0.3825	0.0000	0.0000
34	851.55	35.5	5.7543	3.8826	1.5811	0.3877	0.0000	0.0000
35	895.21	37.3	5.1190	3.4863	1.4754	0.3881	0.0000	0.0000
36	938.88	39.1	4.5621	3.1326	1.3719	0.3842	0.0000	0.0000
37	982.55	40.9	4.0723	2.8167	1.2719	0.3770	0.0000	0.0000
38	1026.22	42.8	3.6404	2.5343	1.1765	0.3670	0.0000	0.0000
39	1069.89	44.6	3.2586	2.2817	1.0861	0.3550	0.0000	0.0000
40	1113.55	46.4	2.9204	2.0556	1.0010	0.3414	0.0000	0.0000
41	1157.22	48.2	2.6202	1.8530	0.9212	0.3267	0.0000	0.0000
42	1200.89	50.0	2.3533	1.6713	0.8469	0.3113	0.0000	0.0000
43	1244.56	51.9	2.1155	1.5083	0.7777	0.2955	0.0000	0.0000
44	1288.23	53.7	1.9034	1.3619	0.7137	0.2796	0.0000	0.0000
45	1331.89	55.5	1.7140	1.2304	0.6544	0.2637	0.0000	0.0000
46	1375.56	57.3	1.5445	1.1121	0.5997	0.2482	0.0000	0.0000
47	1419.23	59.1	1.3928	1.0056	0.5493	0.2330	0.0000	0.0000
48	1462.90	61.0	1.2568	0.9097	0.5029	0.2183	0.0000	0.0000
49	1506.57	62.8	1.1347	0.8233	0.4603	0.2042	0.0000	0.0000
50	1550.23	64.6	1.0251	0.7455	0.4211	0.1906	0.0000	0.0000

STATION 2

X= 27295.0 TEMP= 20.00 Q= 493.10

AVG. VALUES: B=1372.79 Z= 20.12 U= 0.018 EX= 150.245 BETA= 0.0010 KD=0.0000E 00

NO.	TIME(HRS.	:DAYS)	0.00	0.20	0.40	0.60	0.80	1.00
1	91.85	3.8	1.3215	0.0793	0.0000	0.0000	0.0000	0.0000
2	105.63	4.4	2.5017	0.2022	0.0002	0.0000	0.0000	0.0000
3	119.40	5.0	4.1161	0.4233	0.0009	0.0000	0.0000	0.0000
4	133.17	5.5	6.1004	0.7669	0.0028	0.0000	0.0000	0.0000
5	146.94	6.1	8.3462	1.2450	0.0072	0.0000	0.0000	0.0000
6	160.71	6.7	10.7215	1.8549	0.0157	0.0000	0.0000	0.0000
7	174.49	7.3	13.1054	2.5813	0.0307	0.0000	0.0000	0.0000
8	188.26	7.8	15.3675	3.3984	0.0543	0.0001	0.0000	0.0000
9	202.03	8.4	17.4338	4.2740	0.0889	0.0002	0.0000	0.0000
10	215.80	9.0	19.2618	5.1755	0.1363	0.0005	0.0000	0.0000
11	229.57	9.6	20.8171	6.0736	0.1977	0.0010	0.0000	0.0000
12	243.35	10.1	22.0750	6.9425	0.2737	0.0018	0.0000	0.0000
13	257.12	10.7	23.0302	7.7599	0.3642	0.0031	0.0000	0.0000
14	270.89	11.3	23.6978	8.5079	0.4683	0.0050	0.0000	0.0000
15	284.66	11.9	24.1078	9.1739	0.5845	0.0077	0.0000	0.0000
16	298.43	12.4	24.2966	9.7512	0.7110	0.0114	0.0000	0.0000
17	312.21	13.0	24.3015	10.2379	0.8456	0.0163	0.0001	0.0000
18	325.98	13.6	24.1568	10.6362	0.9859	0.0224	0.0001	0.0000
19	339.75	14.2	23.8929	10.9510	1.1295	0.0300	0.0002	0.0000
20	387.18	16.1	22.3945	11.4914	1.6192	0.0679	0.0009	0.0000
21	434.62	18.1	20.4619	11.4132	2.0457	0.1238	0.0027	0.0000
22	482.06	20.1	18.4478	10.9625	2.3751	0.1935	0.0062	0.0000
23	529.49	22.1	16.5136	10.3097	2.5990	0.2701	0.0119	0.0000
24	576.93	24.0	14.7266	9.5632	2.7265	0.3468	0.0200	0.0000
25	624.36	26.0	13.1084	8.7894	2.7738	0.4181	0.0303	0.0000
26	671.80	28.0	11.6592	8.0275	2.7592	0.4800	0.0424	0.0000
27	719.23	30.0	10.3691	7.2997	2.6956	0.5306	0.0555	0.0000
28	766.67	31.9	9.2244	6.6176	2.5994	0.5692	0.0691	0.0000
29	814.10	33.9	8.2103	5.9865	2.4806	0.5960	0.0825	0.0000
30	861.54	35.9	7.3124	5.4073	2.3477	0.6120	0.0951	0.0000
31	908.98	37.9	6.5174	4.8791	2.2072	0.6185	0.1066	0.0000
32	956.41	39.9	5.8132	4.3993	2.0639	0.6167	0.1165	0.0000
33	1003.85	41.8	5.1890	3.9649	1.9215	0.6082	0.1249	0.0000
34	1051.28	43.8	4.6352	3.5723	1.7823	0.5942	0.1316	0.0000
35	1098.72	45.8	4.1436	3.2181	1.6483	0.5759	0.1366	0.0000
36	1146.15	47.8	3.7067	2.8989	1.5205	0.5544	0.1401	0.0000
37	1193.59	49.7	3.3182	2.6114	1.3996	0.5306	0.1420	0.0000
38	1241.03	51.7	2.9723	2.3526	1.2860	0.5052	0.1426	0.0000
39	1288.46	53.7	2.6640	2.1197	1.1799	0.4790	0.1420	0.0000
40	1335.90	55.7	2.3892	1.9101	1.0811	0.4524	0.1404	0.0000
41	1383.33	57.6	2.1439	1.7216	0.9894	0.4258	0.1379	0.0000
42	1430.77	59.6	1.9248	1.5519	0.9047	0.3996	0.1347	0.0000
43	1478.20	61.6	1.7290	1.3993	0.8265	0.3740	0.1308	0.0000
44	1525.64	63.6	1.5538	1.2619	0.7546	0.3493	0.1265	0.0000
45	1573.07	65.5	1.3971	1.1383	0.6885	0.3255	0.1218	0.0000
46	1620.51	67.5	1.2567	1.0270	0.6278	0.3028	0.1168	0.0000
47	1667.95	69.5	1.1308	0.9268	0.5723	0.2812	0.1117	0.0000
48	1715.38	71.5	1.0180	0.8365	0.5214	0.2608	0.1065	0.0000
49	1762.82	73.5	0.9168	0.7552	0.4749	0.2415	0.1012	0.0000
50	1810.25	75.4	0.8259	0.6820	0.4324	0.2234	0.0960	0.0000

STATION 3
X= 45472.0 TEMP= 20.00 G= 493.10
AVG. VALUES: B=1503.26 Z= 20.34 U= 0.016 EX= 143.641 BETA= 0.0010 KD=0.0000E 00

NO.	TIME(HRS.	: DAYS)	0.00	0.20	0.40	0.60	0.80	1.00
1	222.49	9.3	1.3285	0.3341	0.0062	0.0000	0.0000	0.0000
2	244.94	10.2	2.0943	0.5949	0.0156	0.0000	0.0000	0.0000
3	267.38	11.1	3.0153	0.9503	0.0333	0.0002	0.0000	0.0000
4	289.83	12.1	4.0439	1.3947	0.0629	0.0004	0.0000	0.0000
5	312.27	13.0	5.1251	1.9129	0.1077	0.0011	0.0000	0.0000
6	334.71	13.9	6.2056	2.4834	0.1698	0.0022	0.0000	0.0000
7	357.16	14.9	7.2402	3.0826	0.2507	0.0043	0.0000	0.0000
8	379.60	15.8	8.1952	3.6871	0.3500	0.0077	0.0000	0.0000
9	402.05	16.8	9.0490	4.2770	0.4666	0.0127	0.0001	0.0000
10	424.49	17.7	9.7901	4.8360	0.5980	0.0197	0.0002	0.0000
11	446.93	18.6	10.4152	5.3524	0.7413	0.0292	0.0003	0.0000
12	469.38	19.6	10.9264	5.8187	0.8930	0.0412	0.0006	0.0000
13	491.82	20.5	11.3298	6.2305	1.0497	0.0561	0.0010	0.0000
14	514.27	21.4	11.6335	6.5863	1.2081	0.0739	0.0016	0.0000
15	536.71	22.4	11.8468	6.8866	1.3651	0.0945	0.0024	0.0000
16	559.16	23.3	11.9794	7.1332	1.5181	0.1178	0.0034	0.0001
17	581.60	24.2	12.0409	7.3292	1.6650	0.1435	0.0048	0.0001
18	604.04	25.2	12.0404	7.4781	1.8040	0.1714	0.0066	0.0002
19	626.49	26.1	11.9864	7.5837	1.9337	0.2011	0.0087	0.0002
20	685.51	28.6	11.6439	7.6843	2.2244	0.2849	0.0163	0.0006
21	744.52	31.0	11.0993	7.5831	2.4380	0.3716	0.0270	0.0014
22	803.54	33.5	10.4377	7.3400	2.5768	0.4549	0.0406	0.0028
23	862.56	35.9	9.7183	7.0029	2.6491	0.5302	0.0563	0.0048
24	921.58	38.4	8.9815	6.6085	2.6654	0.5945	0.0735	0.0075
25	980.60	40.9	8.2540	6.1837	2.6366	0.6462	0.0912	0.0111
26	1039.62	43.3	7.5526	5.7479	2.5728	0.6850	0.1087	0.0153
27	1098.64	45.8	6.8878	5.3149	2.4830	0.7113	0.1252	0.0201
28	1157.66	48.2	6.2650	4.8939	2.3748	0.7260	0.1403	0.0253
29	1216.67	50.7	5.6867	4.4908	2.2544	0.7306	0.1535	0.0308
30	1275.69	53.2	5.1534	4.1095	2.1268	0.7263	0.1647	0.0363
31	1334.71	55.6	4.6639	3.7521	1.9959	0.7147	0.1736	0.0417
32	1393.73	58.1	4.2166	3.4193	1.8648	0.6972	0.1804	0.0468
33	1452.75	60.5	3.8090	3.1112	1.7357	0.6750	0.1850	0.0516
34	1511.77	63.0	3.4384	2.8272	1.6103	0.6492	0.1877	0.0558
35	1570.79	65.4	3.1023	2.5663	1.4898	0.6209	0.1886	0.0595
36	1629.81	67.9	2.7979	2.3274	1.3750	0.5909	0.1879	0.0626
37	1688.82	70.4	2.5225	2.1091	1.2664	0.5598	0.1857	0.0651
38	1747.84	72.8	2.2736	1.9100	1.1642	0.5283	0.1824	0.0670
39	1806.86	75.3	2.0488	1.7288	1.0685	0.4969	0.1780	0.0683
40	1865.88	77.7	1.8460	1.5641	0.9793	0.4660	0.1729	0.0690
41	1924.90	80.2	1.6631	1.4145	0.8964	0.4358	0.1670	0.0692
42	1983.92	82.7	1.4982	1.2789	0.8197	0.4065	0.1607	0.0689
43	2042.94	85.1	1.3495	1.1558	0.7488	0.3784	0.1540	0.0682
44	2101.95	87.6	1.2156	1.0443	0.6834	0.3516	0.1471	0.0672
45	2160.97	90.0	1.0950	0.9435	0.6233	0.3261	0.1400	0.0658
46	2219.99	92.5	0.9863	0.8522	0.5680	0.3019	0.1328	0.0641
47	2279.01	95.0	0.8885	0.7697	0.5173	0.2791	0.1257	0.0622
48	2338.03	97.4	0.8004	0.6951	0.4709	0.2577	0.1187	0.0601
49	2397.05	99.9	0.7210	0.6277	0.4285	0.2377	0.1118	0.0579
50	2456.07	102.3	0.6496	0.5667	0.3897	0.2190	0.1051	0.0556

STATION 4

X= 62292.0 TEMP= 20.00 Q= 493.10

AVG. VALUES: B=1680.15 Z= 17.47 U= 0.017 EX= 141.098 BETA= 0.0010 KD=0.0000E 00

NO.	TIME(HRS.	: DAYS)	0.00	0.20	0.40	0.60	0.80	1.00
1	355.60	14.8	1.1826	0.4855	0.0350	0.0005	0.0000	0.0000
2	381.63	15.9	1.6873	0.7367	0.0634	0.0012	0.0000	0.0000
3	407.65	17.0	2.2709	1.0472	0.1056	0.0025	0.0000	0.0000
4	433.68	18.1	2.9130	1.4100	0.1636	0.0048	0.0000	0.0000
5	459.70	19.2	3.5905	1.8151	0.2389	0.0086	0.0001	0.0000
6	485.72	20.2	4.2809	2.2502	0.3317	0.0142	0.0002	0.0000
7	511.75	21.3	4.9635	2.7022	0.4413	0.0223	0.0004	0.0000
8	537.77	22.4	5.6210	3.1587	0.5661	0.0332	0.0007	0.0000
9	563.79	23.5	6.2397	3.6086	0.7036	0.0473	0.0011	0.0000
10	589.82	24.6	6.8092	4.0422	0.8513	0.0648	0.0018	0.0000
11	615.84	25.7	7.3227	4.4519	1.0060	0.0858	0.0028	0.0001
12	641.86	26.7	7.7762	4.8317	1.1646	0.1103	0.0042	0.0001
13	667.89	27.8	8.1678	5.1775	1.3243	0.1382	0.0060	0.0002
14	693.91	28.9	8.4979	5.4867	1.4823	0.1691	0.0083	0.0003
15	719.94	30.0	8.7679	5.7579	1.6362	0.2029	0.0111	0.0004
16	745.96	31.1	8.9805	5.9906	1.7840	0.2389	0.0145	0.0006
17	771.98	32.2	9.1390	6.1855	1.9240	0.2768	0.0186	0.0009
18	798.01	33.3	9.2472	6.3438	2.0547	0.3160	0.0233	0.0012
19	824.03	34.3	9.3092	6.4671	2.1752	0.3562	0.0286	0.0017
20	882.52	36.8	9.3015	6.6274	2.4054	0.4469	0.0427	0.0032
21	941.02	39.2	9.1278	6.6475	2.5772	0.5345	0.0596	0.0054
22	999.51	41.6	8.8302	6.5545	2.6919	0.6148	0.0784	0.0084
23	1058.01	44.1	8.4445	6.3742	2.7541	0.6849	0.0984	0.0122
24	1116.50	46.5	8.0004	6.1294	2.7701	0.7430	0.1188	0.0169
25	1175.00	49.0	7.5217	5.8393	2.7472	0.7883	0.1387	0.0222
26	1233.49	51.4	7.0266	5.5200	2.6922	0.8210	0.1575	0:0281
27	1291.99	53.8	6.5292	5.1844	2.6120	0.8415	0.1746	0.0344
28	1350.48	56.3	6.0397	4.8423	2.5126	0.8508	0.1896	0.0409
29	1408.97	58.7	5.5657	4.5017	2.3993	0.8502	0.2023	0.0473
30	1467.47	61.1	5.1122	4.1683	2.2767	0.8410	0.2125	0.0535
31	1525.96	63.6	4.6826	3.8462	2.1495	0.8246	0.2203	0.0594
32	1584.46	66.0	4.2787	3.5383	2.0178	0.8022	0.2256	0.0648
33	1642.95	68.5	3.9015	3.2466	1.8871	0.7750	0.2287	0.0696
34	1701.45	70.9	3.5510	2.9721	1.7584	0.7443	0.2296	0.0737
35	1759.94	73.3	3.2269	2.7154	1.6331	0.7110	0.2286	0.0771
36	1818.44	75.8	2.9283	2.4764	1.5123	0.6758	0.2260	0.0798
37	1876.93	78.2	2.6541	2.2550	1.3969	0.6397	0.2219	0.0817
38	1935.43	80.6	2.4029	2.0505	1.2873	0.6032	0.2165	0.0830
39	1993.92	83.1	2.1734	1.8622	1.1838	0.5668	0.2101	0.0836
40	2052.41	85.5	1.9642	1.6894	1.0866	0.5309	0.2030	0.0836
41	2110.91	88.0	1.7737	1.5311	0.9958	0.4959	0.1951	0.0831
42	2169.40	90.4	1.6007	1.3864	0.9111	0.4620	0.1869	0.0820
43	2227.90	92.8	1.4436	1.2543	0.8325	0.4294	0.1783	0.0805
44	2286.39	95.3	1.3013	1.1341	0.7598	0.3982	0.1695	0.0786
45	2344.89	97.7	1.1724	1.0247	0.6926	0.3686	0.1606	0.0765
46	2403.38	100.1	1.0558	0.9253	0.6307	0.3406	0.1518	0.0740
47	2461.88	102.6	0.9504	0.8351	0.5738	0.3142	0.1430	0.0713
48	2520.37	105.0	0.8553	0.7533	0.5216	0.2894	0.1345	0.0685
49	2578.87	107.5	0.7694	0.6793	0.4737	0.2662	0.1261	0.0656
50	2637.36	109.9	0.6920	0.6123	0.4300	0.2446	0.1180	0.0626